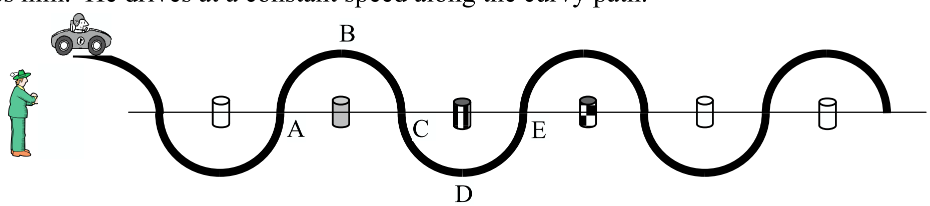
Tutorial #8: **Curviness**

Intuitive Quantum Physics Name:____________________________

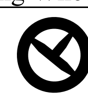
We've spent the last three tutorials developing ideas about energy and probability, building our toolbox of ideas and skills we'll use in the last unit of the course. Today's tutorial deals with the curviness of graphs; for the time being, we'll set aside our ideas about energy and probability, and think about some general characteristics of graphs.

I. Driving Down a Function

Ed takes a driving course in which he has to drive his car around pylons as shown below as Rand watches him. He drives at a constant speed along the curvy path.



Ed's steering wheel has a bar that points straight up when the wheel is not turned. In the table below, record what the nearest pylon is, whether Ed is to the left or right of the pylon according to Rand and the center line, and draw a picture of the orientation of Ed's steering wheel for each part of the path. We've filled in the first row for you.

	Nearest Pylon?	Left or Right of Pylon?	Steering Wheel Drawing
A to B	Solid	Left	
B to C			
C to D			
D to E			

A. At each of points A, B, C, D, and E, is his steering wheel turned? If so, which way? Explain how you can tell.

B. Tell the story of how Ed's steering wheel is turning as he drives down the road. Use words and steering wheel pictures.

C. As Ed drives down the road, does the bar on his steering wheel point *towards* or *away from* the nearest pylon?

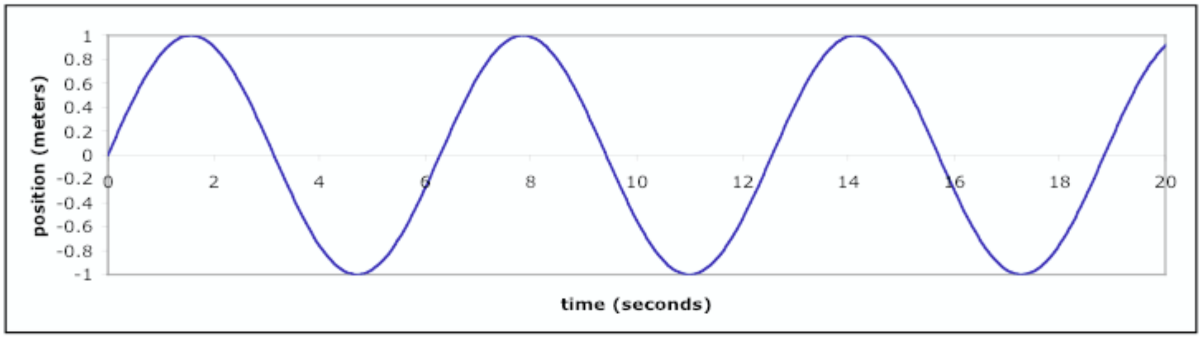
D. Is this true for all the pylons?

E. How is Ed’s steering wheel oriented when he is equally distant from two pylons?

If we imagine a curve on a graph, we can imagine moving from left to right along that curve.
We will use the phrase ***driving down the function*** to describe moving along the graph of a function.

II. S-functions

The graph below is a story graph of the position of an object.



A. Answer the next four questions individually, and then discuss your answers with your group.

1. Where is the object after 10 seconds have elapsed?

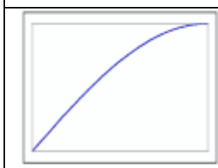
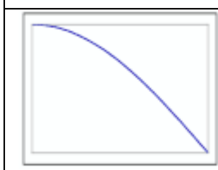
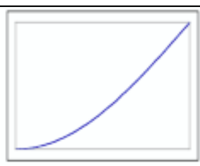
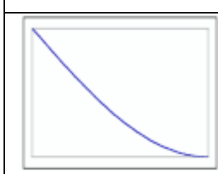
2. When is the object at position 0.6 meters the third time?

3. Describe, in words, at least two physical systems represented by this graph.

4. If you had to predict, what do you think this function would do at times not shown on the graph (at $t > 20$ s, for example)? Is this consistent with the physical systems you just described in 3?

B. The boxes below reproduce small sections of the graph from the previous page. One-by-one, consider just these sections of the graph, and do the following:

- Draw a circle around a part of the curve from which this section could have been taken and label it with the number from the section picture below.
- Determine whether the position of the object is positive or negative.
- Determine the direction that the object is moving. How can you tell? Make sure your group agrees on the perspective you're using to address this question.
- As you "drive down the function" in just this section, is your steering wheel pointed left or right? Draw a picture of it.

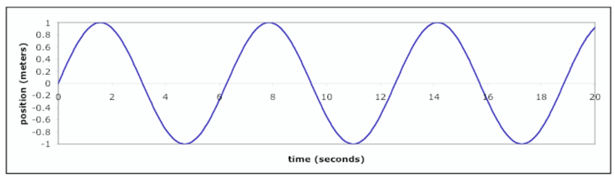
	Position of object + or -?	Direction object is moving	Steering wheel drawing
 1			
 2			
 3			
 4			

C. As you travel from left to right down a function, you can have the vertical bar on your steering wheel pointed either to the left or the right. When the steering wheel is pointed to the right, we say that the graph is *curving down,* or that the curviness is negative.

1. In the sections you analyzed in part B, where is the curviness negative?

2. How are *curving down* and *moving down* different? Support your answers with examples from part B.

D. Compare the *curviness* of the function on page 3 (reproduced at right) to the *value* of the function. (The value of the function at $t = 7$ seconds, for example, is approximately 0.6 cm)



1. When the *value* of the function is positive, is the curviness positive or negative? Support your answers with examples from the sections you looked at in part B.

2. When the *value* of the function is negative, is the curviness positive or negative? Again, support your answers with examples you looked at in part B.

3. We will call a function that has the behavior described in this section an ***s-function***. Roger says, "s-functions always curve towards the horizontal axis". Comment on whether or not this is appropriate.

E. Imagine driving down an s-function, and tell the story of your steering wheel, using words and steering wheel pictures.

F. How, if at all, is this different from the story of Ed's steering wheel as he drives through the pylons?

III. Board Meeting

The axes below represent a picture graph of ψ, the wave function. In lecture, we defined the wave function as the wave that describes things that have both wave and particle behaviors. One point is given. Draw three *substantially* different functions that go through this point (use different color markers if available):

- Two of these functions should be s-functions.
- One of these functions should not be an s-function.

A. Compare the two s-functions you have drawn with each other. What features make them s-functions? What features make them substantially different from each other?

B. Compare the s-functions you have drawn with the non-s-function. What features make your non-s-function not an s-function?

IV. The curviness at a particular point

As a group, use the curviness tool transparency and the handout to fill out the following tables for the driving course curve and the s-function. To find the curviness at a given point, choose the circle that best matches the function at that point.

A. First, consider the picture graph of Ed's position vs. x.

Driving Course Function					
When x=	Value of function + or -	Position =	Curviness + or -	Which circle?	Steering wheel drawing
0.5					
1					
1.5					
2.75					
3.75					

B. Next, look at the picture graph of ψ vs. x, an s-function.

s-function					
When x=	Value of function + or -	ψ =	Curviness + or -	Which circle?	Steering wheel drawing
0.5					
1					
1.5					
2.75					
3.75					

C. On the transparency, which circle has the highest curviness? The lowest? How do you know? Use the steering wheel analogy.

D. On the graph of Ed’s position vs. x, how, if at all, does the curviness change as you move along the function?

E. On the graph of ψ vs. x, how, if at all, does the curviness change as you move along the function?

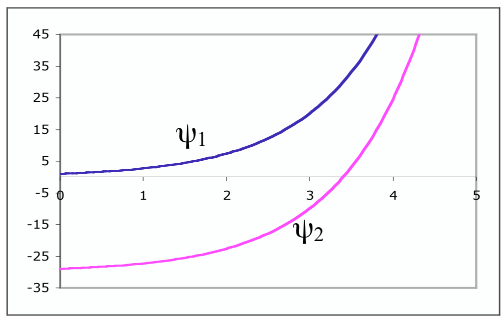
F. Earlier, you told the stories of your steering wheel as you drove down the driving course (section I B) and an s-function (section II E). Looking back, do you need to refine your stories? Consider how the curviness changes at different x-values for the driving course and the s-function.

V. E-functions

So far, we've looked at the curviness of functions, including ψ, when they curve towards the axis. ψ could also curve away from the axis. We call functions that *always* curve away from the axis *e-functions*.

A. Sketch two possible graphs of ψ with the behavior described above.

B. Consider curves ψ_1 and ψ_2 to the right. If you were to drive along ψ_1,



1. would you be above or below the axis?

2. which direction would you be moving, up or down?

3. which way would your wheel be pointed, left or right? Make a steering wheel drawing.

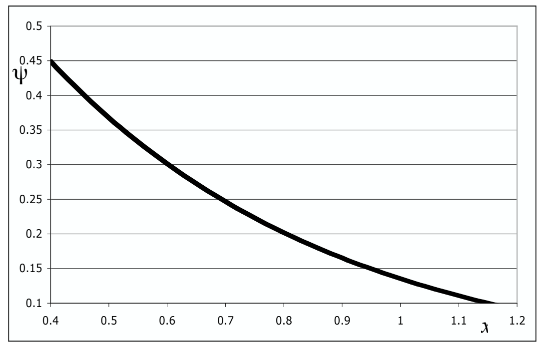
4. would your curviness be positive or negative?

C. Examining the graph of ψ_1, do you expect ψ_1 will ever cross the axis? Explain how you can tell.

D. Are *both*, *one* or *neither* of the graphs e-function(s)? If one, which one? If one or both are not, describe why not.

E. Are e-functions periodic – that is, do they repeat themselves? Explain how you can tell.

F. Consider a portion of a wave function, ψ, at the right. Note the values of the graph. If you were to drive along this ψ,



1. would you be above or below the axis?

2. which direction would you be moving, up or down?

3. which way would your wheel be pointed, left or right? Draw a steering wheel picture.

4. are you curving toward or away from the axis?

5. Does this behave like an e-function? In what ways, if any? Does this behave like an s-function? In what ways, if any?

G. The entire ψ vs. x graph is reproduced on your worksheet. First fill in the chart below, and then answer the questions.

e-function					
When x=	Value of function + or -	ψ =	Curviness + or -	Which circle?	Steering wheel drawing
0.5					
1					
1.5					
2.75					
3.75					

1. Describe how, if at all, the curviness changes for different values of x.

2. Using the steering wheel analogy, give a real-world example of when you would turn your steering wheel as you do when you drive down an e-function.

VI. E-functions and S-functions: A Summary Table.

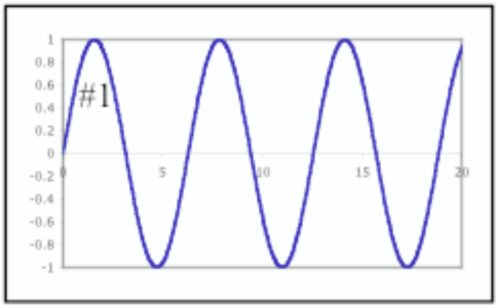
In this section, we're going to synthesize the information from the previous pages into a table that summarizes the traits of s- and e-functions. This page will become an important reference tool for future tutorials.

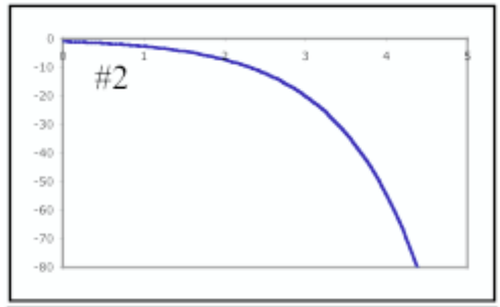
For each of the characteristics below, decide whether or not this applies to s-functions and e-functions. Provide examples from this tutorial (sketches, brief descriptions, etc.) to support your answer.

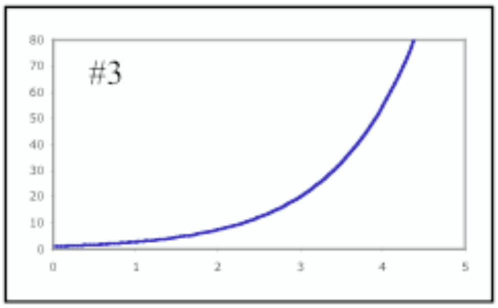
Characteristic	s-functions	e-functions
Can cross the axis	Yes No Example:	Yes No Example:
Is periodic	Yes No Example:	Yes No Example:
Curves towards the axis	Yes No Example:	Yes No Example:
Curves away from the axis	Yes No Example:	Yes No Example:

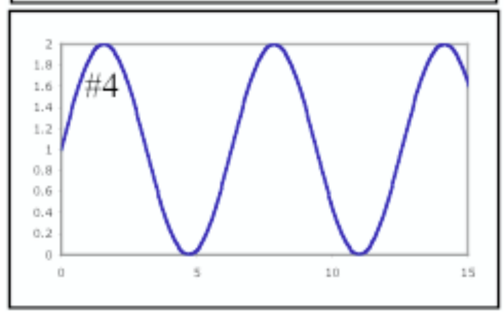
VII. Board Meeting: Classifying Functions

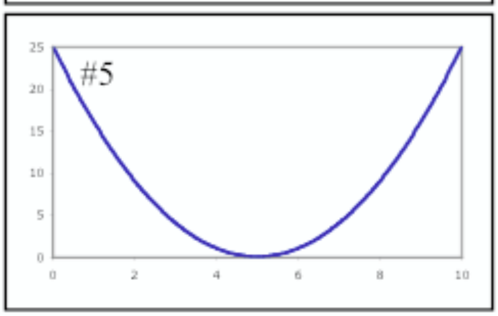
Each of the graphs shown below shows a *single* function. A single function may be either an s-function, an e-function, or neither. Which of the following graphs are e-functions? s-functions? neither? Explain your reasoning for each one. Your instructor will choose groups to discuss each of the graphs during the board meeting.

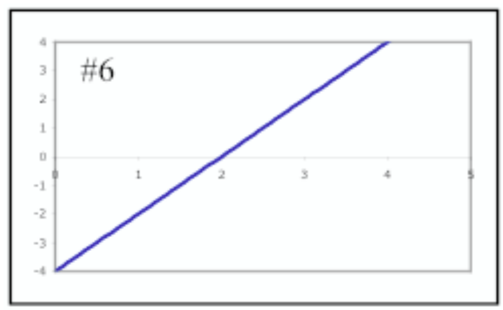


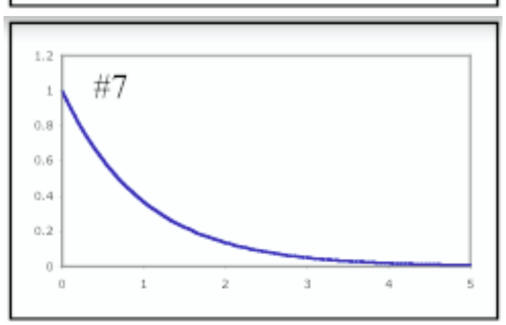


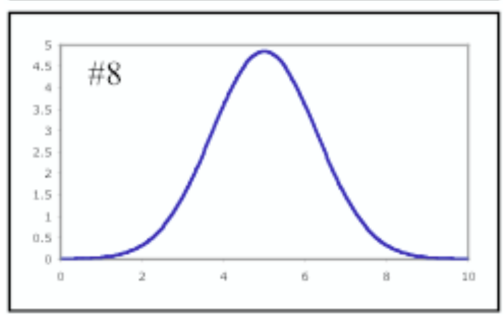


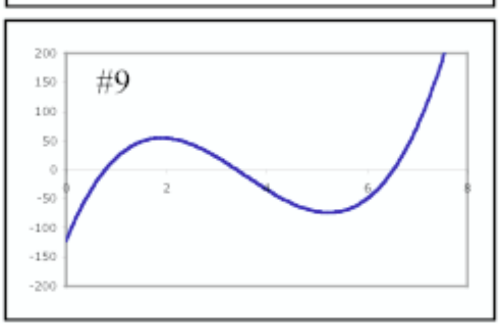


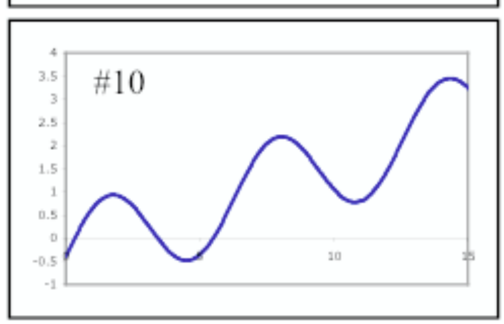






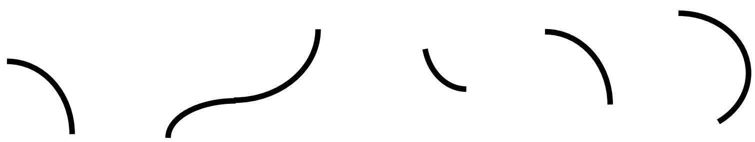




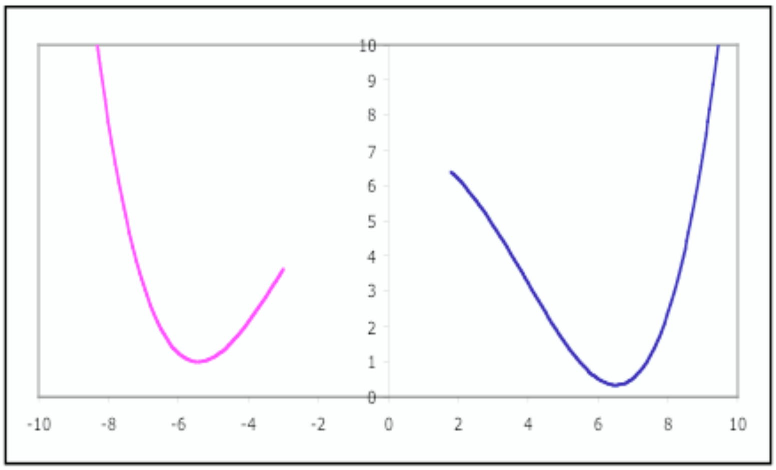


VIII. Stitching Functions Together

A. Imagine that you are a road contractor, and you have to connect the small sections of roads into one long road for a subdivision. The cheap way to connect road segments is using short, straight lines. More expensive, but easier to drive on, is to connect each segment smoothly, with no kinks. Connect the following road sections the cheap way and the expensive way.



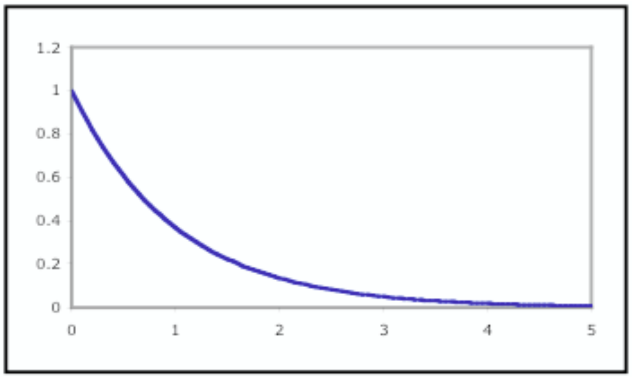
B. Similarly, when two functions connect, they have to connect smoothly without any kinks. When two functions are properly connected, we say they are *stitched together*. Stitch together the two functions to the right.

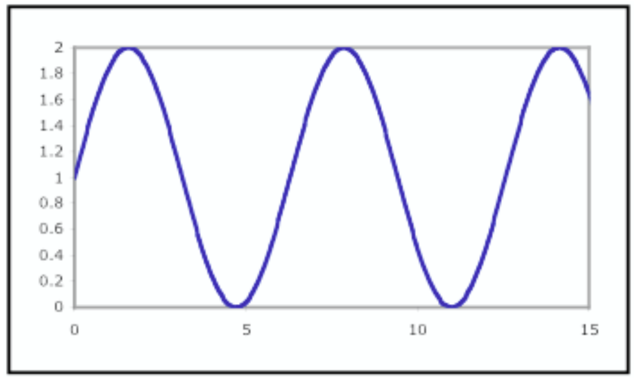


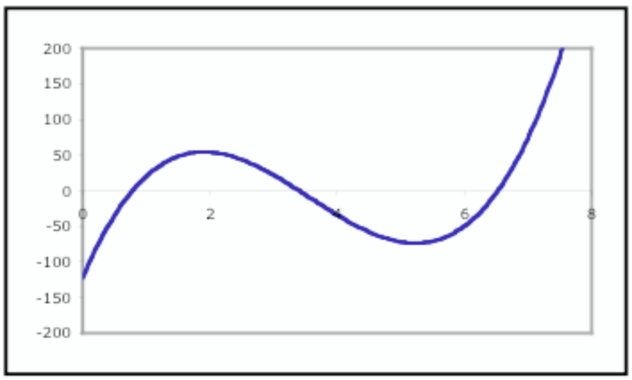
1. Does the segment you drew go higher than the left edge of the function on the right? Why or why not?

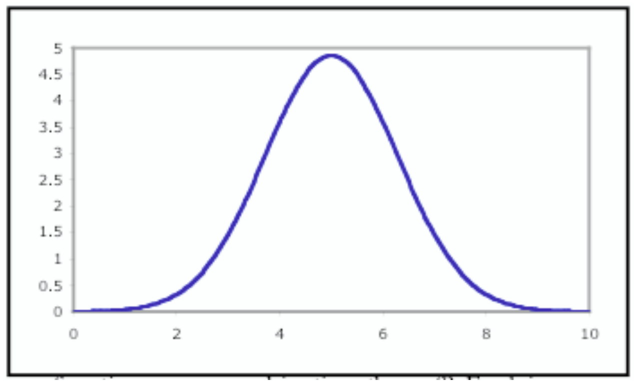
C. We created the above function by stitching together several s-functions and e-functions. On the graph above, identify where the function behaves like an s-function, and where the function behaves like an e-function. Explain your reasoning.

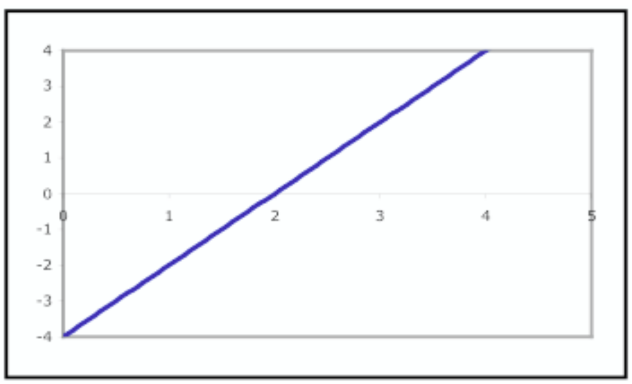
D. Some of the graphs from the board meeting are reproduced here. For each graph, imagine that it is created by stitching together e- and s-functions. Identify where it behaves like an s-function, like an e-function, or like neither.

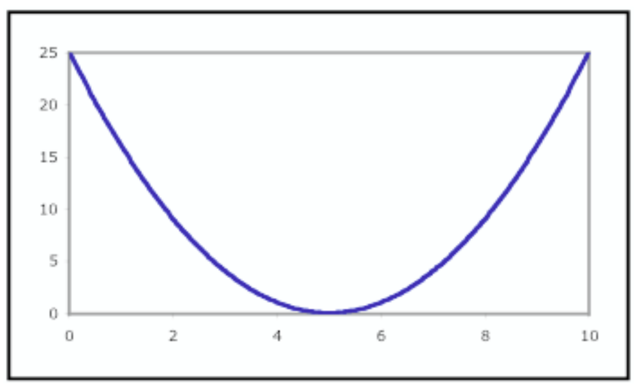












Are there functions that do not behave like s-functions, e-functions, or a combination thereof? Explain your reasoning.

Since there's no board meeting to conclude this tutorial, check your reasoning in the final two sections with your instructor before leaving.