Center for Physics Education
Department of Physics
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Research as a guide for curriculum development:

An excerpt from *Tutorials in Introductory Physics*†

Changes in energy and momentum

Workshop for New Physics and Astronomy Faculty
College Park, Maryland
November 2013

† Lillian C. McDermott, P.S. Shaffer, and the Physics Education Group, University of Washington, Seattle, WA, First Edition, Prentice Hall Inc., 2002.

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Preface

Tutorials in Introductory Physics is a set of instructional materials intended to supplement the lectures and textbook of a standard introductory physics course. The emphasis in the tutorials is on the development of important physical concepts and scientific reasoning skills, not on solving the standard quantitative problems found in traditional textbooks.

There is increasing evidence that after instruction in a typical course, many students are unable to apply the physics formalism that they have studied to situations that they have not expressly memorized. In order for meaningful learning to occur, students need more assistance than they can obtain through listening to lectures, reading the textbook, and solving standard quantitative problems. It can be difficult for students who are studying physics for the first time to recognize what they do and do not understand and to learn to ask themselves the types of questions necessary to come to a functional understanding of the material. *Tutorials in Introductory Physics* provides a structure that promotes the active mental engagement of students in the process of learning physics. Questions in the tutorials guide students through the reasoning necessary to construct concepts and to apply them in real-world situations. The tutorials also provide practice in interpreting various representations (*e.g.*, verbal descriptions, diagrams, graphs, and formulas) and in translating back and forth among them. For the most part, the tutorials are intended to be used after concepts have been introduced in the lectures and the laboratory, although many can serve to introduce the topic as well.

The tutorials comprise an integrated system of pretests, worksheets, homework assignments, and post-tests. The tutorial sequence begins with a pretest, which is usually on material already presented in lecture or textbook but not yet covered in tutorial. The pretests help students identify what they do and not understand about the material and what they are expected to learn in the upcoming tutorial. They also inform the instructors about the level of student understanding. The worksheets, which consist of carefully sequenced tasks and questions, provide the structure for the tutorial sessions. Students work together in small groups, constructing answers for themselves through discussions with one another and with the tutorial instructors. The tutorial instructors do not lecture but ask questions designed to help students find their own answers. The tutorial homework reinforces and extends what is covered in the worksheets. For the tutorials to be most

effective, it is important that the tutorial instructors be well-prepared and that course examinations include questions that emphasize the concepts and reasoning skills developed in the tutorials.

The tutorials are primarily designed for a small class setting but have proved to be adaptable to other instructional environments. The curriculum has been shown to be effective for students in regular and honors sections of calculus-based and algebra-based physics.

The tutorials have been developed through an iterative cycle of: research on the learning and teaching of physics, design of curriculum based on this research, and assessment through rigorous pretesting and post-testing in the classroom. *Tutorials in Introductory Physics* has been developed and tested at the University of Washington and pilot-tested at other colleges and universities.

Comments on the First Edition

Ongoing research has led to modifications to the tutorials and associated homework in the Preliminary Edition of *Tutorials in Introductory Physics*. The First Edition incorporates these changes and also includes several new tutorials on topics covered in the Preliminary Edition. In addition, the First Edition contains a new section with tutorials on topics in hydrostatics, thermal physics, and modern physics.

Acknowledgments

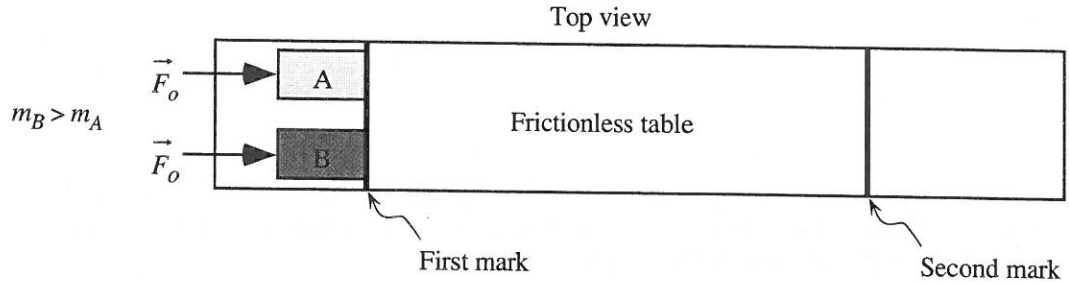
Tutorials in Introductory Physics is the product of close collaboration by many members of the Physics Education Group at the University of Washington. In particular, Paula Heron and Stamatis Vokos, faculty in physics, have played an important role in the development of many tutorials. Significant contributions to the First Edition have also been made by current and former graduate students and post-doctoral research associates: Bradley Ambrose, Andrew Boudreaux, Matt Cochran, Gregory Francis, Stephen Kanim, Christian Kautz, Michael Loverude, Luanna G. Ortiz, Mel Sabella, Rachel Scherr, Mark Somers, John Thompson, and Karen Wosilait. Others, whose work on the Preliminary Edition enriched the First Edition include: Chris Border, Patricia Chastain, Randal Harrington, Pamela Kraus, Graham Oberem, Daryl Pedigo, Tara O'Brien Pride, Christopher Richardson, and Richard Steinberg. Lezlie S. DeWater and Donna Messina, experienced K-12 teachers, have provided many useful insights and suggestions. The assistance of Joan Valles in coordinating the work of the Physics Education Group is deeply appreciated.

The collaboration of other colleagues in the Physics Department has been invaluable. Faculty in the introductory calculus-based sequence, and graduate and undergraduate students who have served as tutorial instructors have made many useful comments. Contributions have also been made by many long-term and short-term visitors to our group. Physics instructors who have pilot-tested the tutorials and have provided valuable feedback over an extended period of time include: John Christopher (University of Kentucky), Romana Crnkovic (Minot State University), William Duxler (Los Angeles Pierce College), Robert Endorf (University of Cincinnati), Gregory Francis (Montana State University), James Freericks and Amy Liu (Georgetown University), Gary Gladding (University of Illinois, Urbana-Champaign), Gregory Kilcup (The Ohio State University), Heidi Mauk (The United States Air Force Academy), Eric Mazur (Harvard University), James Poth (Miami University), and E.F. Redish (University of Maryland).

We thank our editor, Alison Reeves, for her encouragement and advice. We also gratefully acknowledge the support of the National Science Foundation, which has enabled the Physics Education Group to conduct the ongoing, comprehensive program of research, curriculum development, and instruction that has produced *Tutorials in Introductory Physics*. The tutorials have also benefited from the concurrent development of *Physics by Inquiry* (©1996 John Wiley & Sons, Inc.). *Tutorials in Introductory Physics* and *Physics by Inquiry* share a common research base and portions of each have been adapted for the other.

I. Relating forces to changes in kinetic energy and momentum

Two carts, A and B, are initially at rest on a horizontal frictionless table as shown in the top-view diagram below. A constant force of magnitude F_o is exerted on each cart as it travels between two marks on the table. Cart B has a greater mass than cart A.



A. Three students discuss the final momentum and kinetic energy of each cart.

Student 1: *"Since the same force is exerted on both carts, the cart with the smaller mass will move quickly, while the cart with the larger mass will move slowly. The momentum of each cart is equal to its mass times its velocity."*

Student 2: *"This must mean that the speed compensates for the mass and the two carts have equal final momenta."*

Student 3: *"I was thinking about the kinetic energies. Since the velocity is squared to get the kinetic energy but mass isn't, the cart with the bigger speed must have more kinetic energy."*

In the space below, write down whether you agree or disagree with the statements made by each student.

B. Which cart takes longer to travel between the two marks? Explain your reasoning.

C. Use Newton's second law and the definition of acceleration to derive an equation for each cart relating the net force on the cart to the change in velocity of the cart ($\Delta\vec{v}_A$ or $\Delta\vec{v}_B$) and the time interval (Δt_A or Δt_B) that the cart spends between the two marks.

1. Is the quantity $m_A|\Delta\vec{v}_A|$ *greater than, less than,* or *equal to* $m_B|\Delta\vec{v}_B|$? Explain how you can tell.

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For a constant net force, the quantity $\vec{F}_{\text{net}}\,\Delta t$ is called the *impulse* imparted to the object.

2. Is the magnitude of the impulse imparted to cart A *greater than, less than,* or *equal to* the magnitude of the impulse imparted to cart B? Explain your reasoning.

3. Write an equation showing how the impulse imparted to cart A is related to the *change in momentum vector* of cart A ($\Delta\vec{p}_A$), where momentum, denoted by $\vec{p}$, is the product of the mass and velocity of the object.

This relationship is known as the *impulse-momentum theorem.*

4. Is the magnitude of the final momentum of cart A (p_{A_f}) *greater than, less than,* or *equal to* the magnitude of the final momentum of cart B (p_{B_f})? Explain.

D. How does the net work done on cart A ($W_{\text{net},A}$) compare to the net work done on cart B ($W_{\text{net},B}$)? Explain.

Is the kinetic energy of cart A *greater than, less than,* or *equal to* the kinetic energy of cart B after they have passed the second mark?

E. Refer again to the discussion among the three students in part A. Do you agree with your original answer?

If you disagree with any of the students, identify what is incorrect with their statements.

➪ Discuss your answers to parts C and D with a tutorial instructor before continuing.

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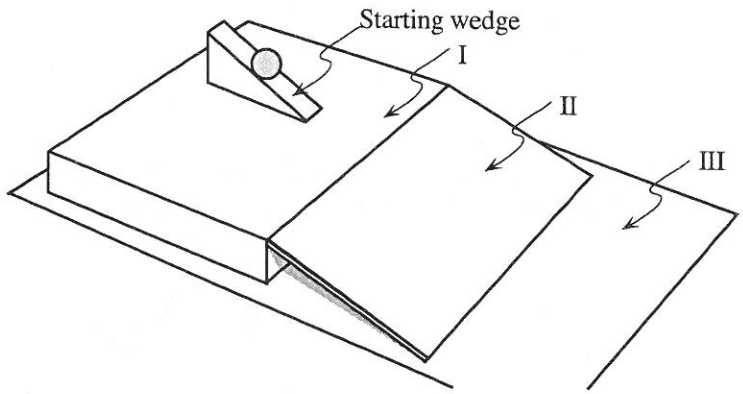
II. Applying the work-energy and impulse-momentum theorems

Obtain a wedge, a ball, a cardboard ramp, and enlargements of the two diagrams below (or sketch them on a large sheet of paper).

(*Note:* It is important that each time the ball is rolled it has the same speed on the level region I. Place a mark halfway up the wedge and release the ball from the mark each time.) Ignore friction and the rotation of the ball.

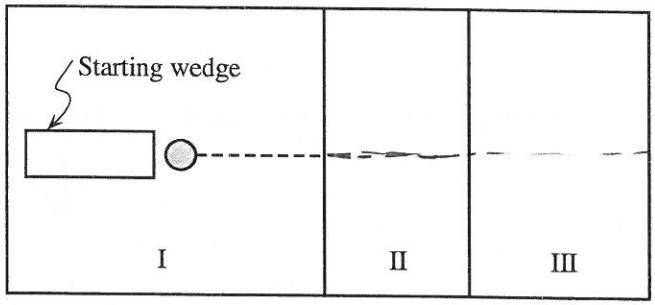


A. Release the ball so that it rolls straight toward the ramp (motion 1).

Observe the motion of the ball.

Sketch the trajectory of the ball on an enlargement for motion 1.

On the enlargement, draw arrows to show the directions of (1) the acceleration of the ball and (2) the net force on the ball while it is on the ramp (*i.e.*, in region II).



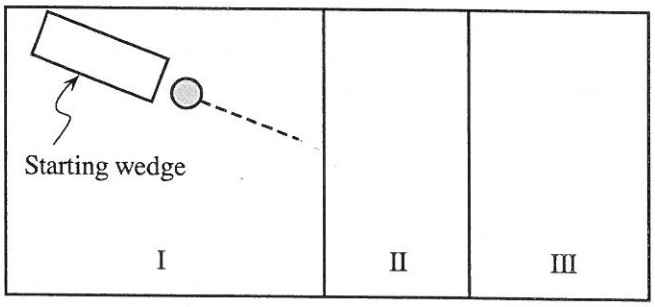
Top view, motion 1

B. Release the ball at an angle to the ramp as shown (motion 2).

Observe the motion of the ball.

Sketch the trajectory of the ball on an enlargement for motion 2.

On the enlargement, draw arrows to show the directions of (1) the acceleration of the ball and (2) the net force on the ball while it is on the ramp (*i.e.*, in region II).



Top view, motion 2

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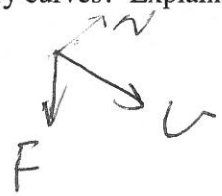
C. How does the direction of the net force on the ball in motion 2 compare to the direction of the net force on the ball in motion 1? Explain.

Same

Is the direction of the acceleration of the ball in motion 2 consistent with the fact that the ball speeds up and its trajectory curves? Explain.



D. How does the change in kinetic energy of the ball in motion 1 compare to the change in kinetic energy of the ball in motion 2?

Same

1. Is your answer consistent with the net work done on the ball in motions 1 and 2? Explain.

$W = \Delta K$

So: The same!

2. How does the final speed of the ball in motion 1 compare to the final speed in motion 2? Explain.

ΔK

The same.

E. For motion 1, draw vectors in region II of the enlargement that represent the momentum of the ball at the top of the ramp and at the bottom of the ramp (*i.e.*, at the top and bottom of region II). Use these vectors to construct the change in momentum vector $\Delta\vec{p}$.

$mv_2 - mv_1 = \Delta p = F \cdot \Delta t$

How is the direction of $\Delta\vec{p}$ related to the direction of the net force on the ball as it rolls down the ramp? Is your answer consistent with the impulse-momentum theorem?

the same.

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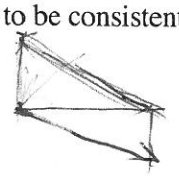
F. For motion 2, draw vectors in region II of the enlargement that represent the initial and the final momentum of the ball. Draw these vectors using the same scale that you used for motion 1 (*i.e.*, the relative lengths should represent the relative magnitudes). Use these vectors to construct the change in momentum vector $\Delta\vec{p}$ for motion 2.

$$\Delta\vec{p}_2 > \Delta\vec{p}_1$$

How should the direction of $\Delta\vec{p}$ compare to the direction of the net force on the ball as it rolls down the ramp? If necessary, modify your diagram to be consistent with the impulse-momentum theorem.



G. Consider the change in momentum vectors you constructed for motions 1 and 2.

1. How do they compare in direction? How do they compare in magnitude?

2. On the basis of your answer above, compare the time that the ball spends on the ramp in motion 1 to the time it spends on the ramp in motion 2. Explain. (*Hint:* Can you use the impulse-momentum theorem to compare the time intervals?)

Is your answer consistent with the trajectory of the ball in each motion? Explain.

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