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Implementation

Purpose of the PCA

To assess essential knowledge that mathematics education research has revealed to be foundational for students' learning and understanding of the central ideas of beginning calculus.

Course Level: What kinds of courses is it appropriate for?

Intro college

Content: What does it test?

Mathematics Content knowledge (rate of change, function, process view of functions, covariational reasoning)

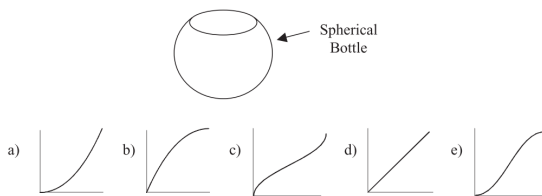
Timing: How long should I give students to take it?

30-45 minutes

Example Questions

Example question from the PCA ([Carlson, Oehrtman, and Engelke, 2010](#)).

Assume that water is poured into a spherical bottle at a constant rate. Which of the following graphs best represents the height of water in the bottle as a function of the amount of water in the bottle?



Access: Where do I get the test?

Email the developers, Marilyn Carlson (marilyn.carlson@asu.edu) or Michael Oehrtman (michael.oehrtman@okstate.edu) to request a copy of the PCA.

Versions and Variations: Which version of the test should I use?

The most recent version of the PCA, version 1, was released in 2010.

Administering: How do I give the test?

- Give it as both a pre- and post-test. This measures student learning.
 - Give the pre-test before you cover relevant course material.
 - Give the post-test at the end of the term.
- Use the whole test, with the original wording and question order. This makes comparisons with other classes meaningful.
- Make the test required, and give credit for completing the test (but not correctness). This ensures maximum participation from your students.
- Tell your students that the test is designed to evaluate the course (not them), and that knowing how they think will help you teach better. Tell them that correctness will not affect their grades (only participation). This helps alleviate student anxiety.
- Refer to the test by a generic title like "Precalculus Survey" to prevent students from looking up the answers.
- For more details, read the **PhysPort Guides** on implementation:
 - **PhysPort Expert Recommendation on Best Practices for Administering Concept Inventories** (www.physport.org/expert/AdministeringConceptInventories/)

Scoring: How do I calculate my students' scores?

- Download the answer key from PhysPort (www.physport.org/key/PCA)

- Each student's score is their percentage correct out of 25 questions.
- See the **PhysPort Expert Recommendation on Best Practices for Administering Concept Inventories** for instructions on calculating normalized gain and effect size (www.physport.org/expert/AdministeringConceptInventories/)
- Use the **PhysPort Assessment Data Explorer** for analysis and visualization of your students' responses (www.physport.org/explore/PCA)

Clusters: Does this test include clusters of questions by topic?

Clusters of questions on the PCA from ([Carlson, Oehrtman, and Engelke, 2010](#)).

TABLE 2
The PCA Taxonomy

<i>Reasoning Abilities</i>	
R1	<i>Process view of function</i> (items 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 20, 22, 23) View a function as a generalized process that accepts input and produces output. Appropriate coordination of multiple function processes
R2	<i>Covariational reasoning</i> (items 15, 18, 19, 24, 25) Coordinate two varying quantities that change in tandem while attending to how the quantities change in relation to each other
R3	<i>Computational abilities</i> (items 1, 3, 4, 10, 11, 14, 16, 17, 21) Identify and apply appropriate algebraic manipulations and procedures to support creating and reasoning about function models
<i>Understandings</i>	
Understand meaning of function concepts	
ME	Function evaluation (items 1, 5, 6, 11, 12, 16, 20)
MR	Rate of change (items 8, 10, 11, 15, 19, 22)
MC	Function composition (items 4, 5, 12, 16, 17, 20, 23)
MI	Function inverse (items 2, 4, 9, 10, 13, 14, 23)
Understand growth rate of function types	
GL	Linear (items 3, 10, 22)
GE	Exponential (item 7)
GR	Rational (items 18, 25)
GN	General non-linear (items 15, 19, 24)
Understand function representations (interpret, use, construct, connect)	
RG	Graphical (items 2, 5, 6, 8, 9, 10, 15, 19, 24)
RA	Algebraic (items 1, 4, 7, 10, 11, 14, 16, 17, 18, 21, 22, 23, 25)
RN	Numerical (items 3, 12, 13)
RC	Contextual (items 3, 4, 7, 8, 10, 11, 15, 17, 18, 20, 22)

Typical Results: What scores are usually achieved?

From ([Carlson, Oehrtman, and Engelke 2010](#)):

The final 25-item version of the PCA has been administered to students upon completion of college algebra and precalculus courses taught in small classes with different teachers at a large public university and nearby community college. The mean score for 550 college algebra students was 6.8 with standard deviation of 3.2. For 902 pre-calculus students, the mean score was 10.2 with standard deviation of 4.1.

While the PCA was not designed for use as a placement exam, it performs reasonably well in this regard. We administered the PCA as a pretest to 248 students entering first-semester introductory calculus courses with six different instructors at our institution and compared final course grades with PCA scores. The most discriminating cut-off score was 50%. Specifically, 77% of the students scoring 13 or higher on the 25-item test passed the first-semester calculus course with a grade of C or better while 60% of the students scoring 12 or lower failed with a D or F or withdrew from the course.

Interpretation: How do I interpret my students' score in light of typical results?

Look at your pre-test scores:

PCA pre-test scores tell you about your students' initial understanding of precalculus concepts, so that you can adjust your teaching based on the knowledge your students start your course with. The developers have also found you can use the PCA as a calculus readiness exam. Recommended cut points for PCA scores (see section on Typical Scores) can help you assess your students preparedness for calculus.

Look at the effect size of the change:

Look at the effect size of the change between your pre- and post-test. This tells you how substantially your pre- and post-test scores differ. Compare your effect size to the ranges given below to find out how substantial the change from pre- to post-test was. For more details, read the **PhysPort Expert Recommendation on Effect Size** (www.physport.org/expert/effectsize)

Effect Size	Cohen's d
Large	~0.8

Medium	~0.5
Small	0.2-0.3

Look at question clusters:

You can also look at the post-test scores for your whole class using the categories or subcategories from the PCA Taxonomy (see section on Clusters) to learn more about which particular concepts your students did well on and which they need help with. This can help you figure out which parts of your teaching worked well, and which parts you could improve in the future.

Resources

Where can I learn more about this test?

R. Meylani and D. Teuscher, [Precalculus Concept Assessment: A predictor of AP Calculus AB and BC scores](#), presented at the Thirty-Third Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education, Reno, Nevada, 2011.

Translations: Where can I find translations of this test in other languages?

Background

Similar Tests

At first blush, the PCA, CCR, and CCI cover very similar topics at a very similar level. However, their emphasis is different, and care should be taken to match the test with your students. The CCR surveys students' understanding of a broad base of mathematics concepts from pre-calculus, including both functions and trigonometry, while the PCA focuses only on the mathematics needed to move into calculus (primarily functions) before calculus instruction. The CCI is designed to test the core concepts of calculus and is aimed at students before and after calculus instruction. Both the PCA and the CCR were developed by the same team of researchers using very similar development methods, and the tests have very a similar structure and feel. The CCI was independently developed by a different team using less robust research methods. If you use these as part of a mathematics placement package or to measure their students' mathematics skills, the CCR is recommended because of the trigonometry and solving equations cluster, though you must pay to use it. Physicists are typically not as interested as mathematicians are in the intricacies of how students understand "function" as a concept, devoid of the physical context, so the PCA and CCI may not be as helpful as the CCR for these purposes.

Research: What research has been done to create and validate the test?

Research Validation: Gold Star ★

This is the highest level of research validation, corresponding to all seven of the validation categories below.

- ✓ Based on research into **student thinking**
- ✓ Studied using **student interviews**
- ✓ Studied using **expert review**
- ✓ Studied using **appropriate statistical analysis**
- ✓ Research conducted **at multiple institutions**
- ✓ Research conducted **by multiple research groups**
- ✓ **Peer-reviewed publication**

Research Overview

The PCA developers did research around the understandings and abilities that are foundational to learning calculus, and developed a taxonomy based on their findings. They used the PCA taxonomy to write open-ended questions and tested them with students. They then interviewed students about their answers and revised the questions. Multiple-choice answers, based on students' written answers, were developed and tested. The cycle of testing the questions with students, interviewing students, and revising the questions and answer choices was repeated eight times. While iteratively developing questions, scoring rubrics were created and

reviewed by experts in math. These rubrics were used to refine the answer choices. Statistical analysis of reliability found reasonable results. The PCA has been used with thousand of students at over 40 institutions and the results published in one peer-reviewed paper.

Developer: Who developed this test?

Marilyn Carlson, Michael Oehrtman, Nicole Engelke

References

- M. Carlson, M. Oehrtman, and N. Engelke, [The Precalculus Concept Assessment: A Tool for Assessing Students' Reasoning Abilities and Understandings](#) Cognition and Instruction **28** (2) 113-145 (2010).
- R. Meylani and D. Teuscher, [Precalculus Concept Assessment: A predictor of AP Calculus AB and BC scores](#), presented at the Thirty-Third Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education, Reno, Nevada , 2011.