

## Some Faculty-consensus (general) Quantum Mechanics topical Learning goals.

The goals below should not be considered a complete list “spanning” a quantum course. They were merely the topics which received the most attention at a (Fall 2017) Quantum learning goals faculty workshop in Boulder. Largely, participants focused on topical-level goals.

All goals should be preceded by: “Students should be able to...”

### 1. Quantum states:

- A. ...calculate normalization constants for quantum states.
- B. ...recognize the difference between an overall phase and a relative phase and the effect that has on measurement outcomes.
- C. ...convert states between different bases (e.g. x-, y-, z-, or energy bases for spin, or between x and Energy, or x and p for position).
- D. ...write the time evolution of a quantum state, given the state at  $t=0$  and the Hamiltonian. (first-level version: state given in energy eigenbasis; second-level version: state not given in energy eigenbasis)
- E. ...distinguish between energy eigenstates, superposition states, and mixed states.

### 2. Observables:

- A. ...determine the possible set of values that could result from a measurement of a given observable (i.e., calculate eigenvalues and recognize they represent possible measurement outcomes)
- B. ...calculate the probability of a measurement outcome given a quantum state (including time dependence when relevant).
- C. ...calculate expectation values for a given observable and given quantum state using multiple methods when appropriate (including time dependence when relevant).
- D. ...distinguish between expectation values, allowed values, most probable value.
- E. ...calculate the uncertainty for an observable given a quantum state.
- F. ...determine the resulting quantum state after a measurement (e.g. after a spin is measured to be up in the x-direction).
- G. ...determine whether two observables can be simultaneously determined.

### 3. Hamiltonian and the Schrodinger Equation:

- A. ...distinguish the time dependent Schrödinger equation from the energy eigen-equation (aka “time independent Schrodinger equation”).
- B. ...write the appropriate Hamiltonian for a given physical context.
- C. ...use the energy eigenvalue equation (aka “time independent Schrödinger equation”) to solve for the energy eigenstates and/or eigenvalues for a given system.
- D. ...identify and apply boundary conditions in a 1-D piecewise-constant potential.
- E. ...sketch a qualitatively correct wave function given a 1D potential (attending to features like the number of nodes, the sign of the curvature, the relative wavelengths and amplitudes).

### 4. Formalism and mathematics:

- A. ...rewrite any complex number between rectangular and polar form.

- B. ...distinguish operators, eigenfunctions (or eigenvectors or eigenstates), and eigenvalues.
- C. ...find the eigenvalues and eigenvectors of given operators.
- D. ...use the technical language of QM correctly (for example: basis, bra and ket, commutator, expectation value, Hamiltonian, Hermitian conjugate, Hilbert Space, operator, probability, projection, uncertainty,...).
- E. ...use the commutator to determine if an observable is a conserved quantity.
- F. ...distinguish between applying an operator to a state and making a measurement.

**5. Widely considered important (but harder to operationalize or directly test)**

- A. ...connect mathematical results to physics<sup>1</sup>
- B. ...use postulates of quantum mechanics to guide problem solving.
- C. ...formulate strategies to solve well-defined problems (regardless of whether they can perform the calculation).
- D. ...be able to move between different representations (for example: bra-ket notation, matrix notation [explicit and/or abstract matrices], wave functions, graphical visualizations, computer code, ...)

...recognize the limits of classical theory and where we need to assume QM

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<sup>1</sup> Math/Physics connections is a common, generic course-level learning goal in any physics course. Here were some concrete examples of math/physics connections in QM:

- students can defend the functional form or behavior of spatial wave functions based on classical intuitions or physical requirements
- students can give qualitative arguments for limiting behaviors of transmission and reflection formulas based on physical or classical intuitions.
- students can productively connect spin-precession formalism to classical or physical top motion, (or, interpret various mathematical limits of Rabi's formula to more classical pictures of classical magnets in fields)
- students can make useful analogies of spin  $\frac{1}{2}$  formalism to more classical photon polarization measurements

**For our students:**

**QM1 and QM2 CU Boulder BROAD COURSE SCALE LEARNING GOALS**

1. **Math/physics connection:** Students should be able to translate a physical description of a junior-level quantum mechanics problem to a mathematical equation necessary to solve it. Students should be able to explain the physical meaning of the formal and/or mathematical formulation of and/or solution to a junior-level QM problem. Students should be able to achieve physical insight through the mathematics of a problem.
2. **Visualize the problem:** Students should be able to sketch the physical parameters of a problem (e.g.,  $\psi$ , potential energy, probability distribution), as appropriate for a particular problem.
3. **Organized knowledge:** Students should be able to articulate the big ideas from each chapter, section, and/or lecture, thus indicating that they have organized their content knowledge. They should be able to filter this knowledge to access the information that they need to apply to a particular physical problem.
4. **Communication.** Students should be able to justify and explain their thinking and/or approach to a problem or physical situation, in either written or oral form.
5. **Problem-solving techniques:** Students should be able to choose and apply the problem-solving technique that is appropriate to a particular problem. This indicates that they have learned the essential features of different problem-solving techniques (eg., separation of variables, power series solutions, operator methods). They should be able to apply these problem-solving approaches to novel contexts (i.e., to solve problems which do not map directly to those in the book), indicating that they understand the essential features of the technique rather than just the mechanics of its application. They should be able to justify their approach for solving a particular problem.
  - ...a. **Approximations:** Students should be able to recognize when approximations are useful, and use them effectively (eg., when the energy is very high, or barrier width very wide,...). Students should be able to indicate how many terms of a series solution must be retained to obtain a solution of a given order.
  - ...b. **Symmetries:** Students should be able to recognize symmetries and be able to take advantage of them in order to choose the appropriate method for solving a problem (eg., when parity allows you to eliminate certain solutions).
  - ...c. **Matrix methods and Dirac notation:** Given a physical situation, students should be able to interpret and compute using Dirac notation, write

down the required matrix equation for Schrodinger's equation, and correctly calculate the answer.

**...d. differential equation methods:** Given a physical situation, students should be able to write down the required (Schrodinger) differential equation, and correctly calculate the answer.

**...e. Metacognition:** Students should be able to justify their choices in problem solving methods (see LG #4 above) verbally or in writing, and explicitly engage in discussion about their thinking and what helped them learn.

6. **Problem-solving strategy:** Students should be able to draw upon an organized set of content knowledge (LG#3), and apply problem-solving techniques (LG#4) to that knowledge in order to organize and carry out long analyses of physical problems. They should be able to connect the pieces of a problem to reach the final solution. They should recognize that wrong turns are valuable in learning the material, be able to recover from their mistakes, and persist in working to the solution even though they don't necessarily see the path to the solution when they begin the problem. Students should be able to articulate what it is that needs to be solved in a particular problem and know when they have solved it.
7. **Expecting and checking solution:** When appropriate for a given problem, students should be able to articulate their expectations for the solution to a problem, such as general shape of the wave function, dependence on coordinate choice, and behavior at large distances. For all problems, students should be able to justify the reasonableness of a solution they have reached, by methods such as checking the symmetry of the solution, looking at limits, relating to cases with known solutions, checking units, dimensional analysis, and/or checking the scale/order of magnitude of the answer.
8. **Intellectual maturity:** Students should accept responsibility for their own learning. They should be aware of what they do and don't understand about physical phenomena and classes of problem. This is evidenced by asking sophisticated, specific questions; being able to articulate where in a problem they experienced difficulty; and take action to move beyond that difficulty.
9. **Build on Earlier Material.** Students should deepen their understanding of Phys 2170 material. I.e., the course should build on earlier material.

## For our students: QM1 CU Boulder TOPICAL LEARNING GOALS

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### McIntyre Ch 1:

You should be able to...

1. ...Describe and derive the basic classical physics of the Stern-Gerlach device, including predicting qualitative motion of uncharged classical objects in a given Stern-Gerlach field
2. ...predict the outcome probabilities and outcome "states" along any given paths of Chained-Stern-Gerlach devices of any arbitrary configuration, including ones oriented in X, Y, Z, or n (tilted by  $\theta$  and  $\phi$ ) directions.
3. ... work backwards along Chained-S-G devices, e.g. given outcomes, decide what S-G orientations were needed or what starting states we began with
4. ...work with basic Dirac notation, including manipulating basic formal expressions (e.g. knowing when term order matters, or which expressions are meaningless), or how to distribute terms involving sums in the bra or ket.
5. ... be able to convert kets to and from bras, normalize kets, find orthogonal kets, compute brackets, and handle the basic complex number manipulations associated with such problems.
6. ... Use Postulate 4 to predict experimental outcomes. This includes working with the formal notation of Postulate 4 and being able to move back and forth between sections of Chained S-G's and the Postulate 4 probability formula.
7. ... generalize to quantum systems beyond spin 1/2 (e.g. doing normalization or use Postulate 4 for spin-1 systems)
8. ... Be able to use matrix notation to compute brackets and express bras and kets

**McIntyre Ch 2:** You should be able to...

**(If you see something you think is MISSING from this list, or that does not BELONG in this list, let me know! )**

1. ....represent an operator in the  $S_z$  basis, given its eigenvalue equations and eigenvectors.
2. ...diagonalize a  $2 \times 2$  matrix (find the eigenvalues, and/or given an eigenvalue, find an eigenvector)
3. ... Use the  $S_x$ ,  $S_y$ , and  $S_z$  matrices and eigenvectors (which will be given on MY crib sheet, Eq 2.38 on page 41) in problems involving S-G problems, probability predictions, or other typical situations.
4. ... Use the  $S_n$  matrix, and eigenvectors (which will be given on MY crib sheet, Eq 2.41 and 2.42 on pp 41-42) to handle chained S-G problems involving  $\hat{N}$  oriented S-G's. This includes understanding how to relate " $\theta$ " in the  $\hat{N}$  picture to " $\theta/2$ " which appears in the  $|+/-\rangle_n$  kets.

5. ... Use postulate 4 to predict probabilities of measurements given any combination of spin 1/2 states and detectors, or work backwards (given the results, to draw conclusions about the measurement device's orientation)
6. ... identify when a matrix is Hermitian, and form a hermitian conjugate of a 2x2 or 3x3 matrix.
7. ... Know the definition of a "projection operator",  $P_n = |+\rangle\langle +|$ , and be able to use it to project components of kets, as well as engage in basic formal dirac notation manipulations ( such as what appears in Eq 2.54, or throughout section 2.2.4)
8. ... Interpret both numerator and denominator of postulate 5, including being able to compute either of those separately given a state and a measurement outcome, and interpret the meaning of Postulate 5 to predict outcome states from chained S-G setups.
9. ... compute "expectation values" of any given operator, interpret this as a sum of possible results\*probabilities (as given e.g. in Eq 2.74, but for any operator).
- 10.... compute the RMS deviation ( $\Delta$ ) for an operator, given a state.
- 11....construct or interpret probability distributions such as McIntyre's figure 2.8 for arbitrary states and measurements.
- 12.... compute the commutator of 2 operators,
- 13.... connect commutation (or non-commutation) to measurements ("compatibility") , uncertainties, and whether 2 operators share common eigen-bases.
- 14.... Know the definition of  $S^2$  (the  $S^2$  operator), and evaluate its effect on spin 1/2 states.
- 15.... apply the rules and postulates of quantum mechanics described above to spin 1 systems in simple cases. (e.g. normalizing spin 1 states, finding brackets, determining orthogonality, computing probabilities using Postulate 4, predicting output states using Postulate 5, or sketching probability distributions as in e.g. Fig 2.13)
- 16.... Generalize to arbitrary system of spin. We will not go too far computationally here - E.g. no diagonalizing of nasty matrices bigger than 2x2 on a test. But e.g. I would expect you to interpret formulas written in n dimensions, "projection operators" in spin-n systems, or compute probabilities in the usual way with Postulate 4 no matter what the dimensionality, at the level shown in e.g McIntyre Chapter 2.8)
- 17.... Apply the Uncertainty principle (Eq 2.98, which will be on MY crib sheet) to arbitrary spin 1/2 operators and states.  
It can be helpful to know (or quickly rederive) the commutation relations in McIntyre Eq 2.96

(Exam 1 takes us through Chapter 2)

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### McIntyre Ch 3 : You should be able to...

1. ...construct a Hamiltonian operator (2x2 or 3x3 matrix) for a spin 1/2 (or 1) particle in an arbitrary B-field
2. ... solve for the "cn" coefficients in the initial state in the  $|E_n\rangle$  basis.
3. ...write down the time dependent solution given initial conditions, and use this to compute probabilities or measurement outcomes or expectation values of any operators of interest at later times. (You should be able to do this even if the field points in a direction other than z-hat, and be able to handle spin-1 as well as spin 1/2)
4. ... qualitatively describe "spin precession", for an \*arbitrary\* magnetic field direction.
5. ... apply Rabi's formula to find "flip probability".
6. ... extend the time-dependent formalism above to "non-spin" systems (neutrino oscillations being one good example, but any 2-state system would work the same)

Note that we did not cover 3.4 in any detail, I will not be testing you on the formulas or formalism in that section.

### McIntyre Ch 4 : You should be able to...

1. ...Use, interpret, and generate 2-particle states in the form used in McIntyre, e.g. his Eq 4.1
2. ... Do standard Quantum calculations with 2-state systems including computing brackets and acting single-particle operators on a state (as e.g. done in Eq 4.2 and 4.3) The key here is to recognize that "1" and "2" states and operators ignore states and operators of the opposite number, and otherwise behave as they always have.
3. ... Given an EPR gedanken experiment setup (like in Fig 4.1) predict how observations at A will correlate with observations at B.
4. ... distinguish "hidden variable" from "quantum" predictions - e.g. be able to decide if a setup will or will not obey the Bell inequalities.

### McIntyre Ch 5 : You should be able to...

1. ...Interpret "probability histograms" (like in Fig 5.2), be able to go back and forth between that representation or an expansion of the form  $|\psi\rangle = \sum c_n |E_n\rangle$  (where you should be able to compute  $c_n$  by evaluating  $\langle E_n|\psi\rangle$  and interpret  $|c_n|^2$  as the probability of finding the energy  $E_n$ , as shown in that histogram.
2. ... know what the x and p operators are in the position representation
3. ... Compute probability of finding a particle within dx of position x using the wave function (i.e. you should know that Prob density =  $|\psi(x)|^2$ , and you should know the difference between "probability density" and "probability")

4. ...be able to form brackets of position-space wavefunctions by forming an integral, and interpreting the bracket as we have in earlier chapters to compute probabilities, or normalize wavefunctions.
5. ... Compute expectation values of given operators in the position space representation by integrating
6. ... write down the time-independent Schrodinger equation as a 2nd order ODE for a given potential function.
7. ... Solve the infinite square well (even if I make small changes, e.g. the location of the starting/ending points of the well, the width of the well, the value of the base of the well)
8. ... use the rules of QM we have developed to compute the time dependence of superposition states of "infinite well"  $|E_n\rangle$  states, find the probabilities of measuring energy for such time dependent superposition states, and find the position probability density for such states.
9. ... know the standard definitions of wavelength, wave number,

**(Exam 2 will take us through this much of Ch 5 - basically everything covered up through the HW due before the exam)**

**For the final, more on Ch 5:**

1. ... know and apply the "boundary conditions" - namely, that wave functions are continuous everywhere,  $d/dx(\text{wave function})$  is continuous unless  $V=\text{infinity}$ , in which case the wave function must vanish)
2. ... Be able to write the "general solution" to Schrodinger's equation in regions where  $V(x)=\text{constant}$
3. ... solve the "finite square well" problem (even if I make small changes, e.g. the location of the starting/ending points of the well, the width of the well, the value of the base or top of the well)
4. ... Sketch qualitatively correct wave functions even when  $V(x)$  is not simple to solve exactly - including getting the sign of the curvature right, the behaviour in "allowed" or "forbidden" regions, application of boundary conditions, getting the relative wavelength right based on KE, and getting the relative amplitudes right based on classical arguments about where the particle spends the most time...  
(Note that I am not going to cover computational solutions to Schrodinger's equation on exams)

**McIntyre Ch 6 : You should be able to...**

1. ... Solve the Schrodinger equation in position space for unbound particles (when  $V=\text{constant}$ ): both the time-independent ( $\exp[i kx]$ ) solutions , knowing what "k" is..., and also the time-dependent solutions.
2. ... Know and use the deBroglie relation for plane waves to relate momentum, wavenumber k, and wavelength.

3. ... know the eigenfunctions of momentum in position space (using the Dirac normalization)
4. ... Be able to use the definitions to compute Fourier transform and Inverse Fourier transform (which will be on the crib sheet). I will remind you of any "Gaussian" formulas if you need them, but I would expect you to be able to integrate delta functions or constant functions on your own...
5. ... Have a qualitative understanding of wave packets, including familiarity with terminology like "envelope", "phase velocity" and "group velocity", and "momentum representation"
6. ... Be able to relate the width of wave packets in position and momentum space.
7. ... Use the position-momentum Heisenberg uncertainty relation (which will be on the crib sheet) to make rough estimates of energy or momentum, like we did in HW or CH 6.3.1
8. ... (As above in item 1) solve the Schrodinger Equation in regions of space where  $V=\text{constant}$ , using "boundary conditions" on the wave function (continuity of  $\psi$  and  $\psi'(x)$ ) to piece together solutions of Schrodinger equation for unbound particles hitting a potential well or barrier
9. ... Interpret the coefficients of plane waves as a measure of "flux" or "flow", thus deciding which (if any) vanish
10. ... Use formulas for T or R (I will give them if you need them) to make qualitative and quantitative predictions of reflection and transmission, in scattering and tunneling situations. This means you know and can interpret/evaluate the "k" values in the different regions, including forbidden ones.
11. Section 6.6 will not be on the exam.

**McIntyre Ch 7 : You should be able to...**

1. ... Recognize situations where you can use the general method of "separation of variables" to simplify partial differential equations, and do so.
2. ... **THIS WILL NOT BE COVERED FOR THE Sp21 FINAL:** Separate the Center of mass from relative motion coordinates in central potential problems. (You should be able to simply write down the CM part of the wave function, which is a 3-D plane wave)
3. ... know and use spherical coordinates including volume integrals
4. ... be able to compute basic commutation relations involving angular momentum operators, including knowing which ones do and do not commute.
5. ... be able to compute or predict experimental outcomes of measurements (including probabilities, and also the "final state") of  $L^2$  or components of  $L$  ( $L_z$ , or  $L_y$ , or  $L_x$ ) for states given in the usual form  $|l, m\rangle$
6. ... solve "particle on a ring" problems, including knowing the energy formula, and being able to predict experimental outcomes of energy as well as  $L_z$ .

You should also be able to add in "time dependence" and compute spatial probability densities for such states.

7. ... solve "particle on a sphere" problems, including knowing the energy formula, and being able to predict experimental outcomes of energy as well as  $L_z$  or  $L^2$ . You should also be able to add in "time dependence" and compute spatial probability densities for such states.
8. ... Know the solution for the  $\Phi(\phi)$  separated solution of wavefunctions (plane waves in  $\phi$ ), including normalization
9. ... **THIS WILL NOT BE COVERED FOR THE Sp21 FINAL:** Know and interpret the standard notation of Legendre functions and Spherical harmonics
10. ... Use the postulates of quantum mechanics to compute probabilities of measurements on angular momentum eigenstates in Dirac notation  $|l,m\rangle$  **Sp 21 we will not require the  $Y_{lm}(\theta,\phi)$  notation here...**

**McIntyre Ch 8 : SP21 WE WILL NOT COVER ANY OF CH8, NONE OF THE BELOW WILL BE ON OUR FINAL** But, for future reference, if you ever look over Ch 8 in the future, your focus might be to...

1. ... know and interpret the standard notation of general solutions to the hydrogen atom,  $\psi_{n,l,m}$ ,
2. ... use the postulates of quantum mechanics to compute probabilities of measurements of energy or angular momentum on hydrogen atom states
3. ... use the standard "time evolution" methods from Ch 3 on hydrogen atom wave functions.
4. ... Answer very basic questions about the spectrum of Hydrogen.

As always, let me know if you think I'm missing some key ideas, I'll add them! If you don't understand what I'm after, or are wondering about some particular topical area and what I might expect from you on an exam - don't hesitate to ask!

## For our students: QM2 (second semester) CU Boulder TOPICAL LEARNING GOALS

First, just an abbreviated summary of QM1 material (McIntyre Ch 1-7) that I expect you to know and use as part of QM2 problem solving.

You should be able to...

From Chapter 1:

1. ... understand and use the postulates of Quantum as outlined at the end of Chapter 1.

From Chapter 2:

2. ... work with basic Dirac notation, including manipulating formal expressions, normalize kets, find orthogonal kets, compute brackets, and handle the basic complex number manipulations associated with such problems.
3. ... Use Postulate 4 to predict experimental outcomes.
4. ... Use Postulate 5 ("collapse") to predict outcome states after measurement outcomes are known
5. ... Be able to compute "expectation values" of any given operator, interpret this as a sum of possible results\*probabilities (as given e.g. in Eq 2.74, but for any operator) but also to compute it as a matrix product (in spin space) or as an integral (in continuous variables like position or momentum)
6. ... connect commutation (or non-commutation) to measurements ("compatibility"), uncertainties, and whether 2 operators share common eigen-bases

Ch 3:

7. ... solve for "cn" coefficients ,given an initial state, in the  $|E_n\rangle$  basis.
8. ...write down the time dependent solution given initial conditions, and use this to compute probabilities or measurement outcomes or expectation values of any operators of interest at later times.

Ch 4: (We will come back to this later in the course, not needed for Exam 1!!)

9. ... Use, interpret, and generate 2-particle states in the form used in McIntyre, e.g. his Eq 4.1
10. ... Do standard Quantum calculations with 2-state systems including computing brackets and acting single-particle operators on a state

Ch 5:

11. ... know what the x and p operators are in the position and momentum representation

12. ... Compute probability of finding a particle within  $dx$  of position  $x$  using the wave function (i.e. you should know that Prob density =  $|\psi(x)|^2$ , and you should know the difference between "probability density" and "probability")
13. ... write down the time-independent Schrodinger equation as a 2nd order ODE for a given potential function, and solve it in familiar cases (like the infinite well)

Ch 6: (Won't be much of this on Exam 1, we have not gotten back to "free particles" yet!)

14. ... Know the position-momentum Heisenberg uncertainty relation (which will be on the crib sheet)
15. ... solve the Schrodinger Equation in regions of space where  $V=\text{constant}$ , using "boundary conditions" on the wave function (continuity of  $\psi$  and  $\psi'(x)$ )

Ch 7:

16. ... be able to compute basic commutation relations involving angular momentum operators, including knowing which ones do and do not commute.
17. ... be able to compute or predict experimental outcomes of measurements (including probabilities, and also the "final state") of  $L^2$  or components of  $L$  ( $L_z$ , or  $L_y$ , or  $L_x$ ) for states given in the usual form  $|l, m\rangle$

**AND, HERE IS WHERE QM2 really begins, so this is where you may want to really begin review/study for QM2:**

18. ... Know and interpret the standard notation of Legendre functions and Spherical harmonics, e.g. know what  $|l, m\rangle$  means and what its eigenvalues are. (This will also be on my crib sheet)
19. ... Use the postulates of quantum mechanics to compute probabilities of measurements on angular momentum eigenstates (either in position representation,  $Y_{lm}(\theta, \phi)$  or dirac notation  $|l, m\rangle$ )

**McIntyre Ch 8 :**

**You should be able to...**

1. ... know and interpret the standard notation of general solutions to the hydrogen atom,  $\psi_{n,l,m}$ , or  $|n, l, m\rangle$
2. ... use the postulates of quantum mechanics to compute probabilities of measurements of energy or angular momentum on hydrogen atom states, including when a measurement partially or wholly "collapses" a state
3. ... know and use the "orthogonormality" relations for  $|n, l, m\rangle$  states to simplify brackets

4. ... use the standard "time evolution" methods from Ch 3 on hydrogen atom wave functions.
5. ... Answer basic questions about the spectrum of Hydrogen.

### **McIntyre Ch 9 : Harmonic Oscillator**

#### **You should be able to...**

1. ... Answer basic questions about the spectrum of the Harmonic oscillator
2. ... use "raising and lowering" operators for HO states, including normalization (see crib sheet!)
3. ... Know and use commutation relations for combinations of  $a$ ,  $a^\dagger$ , and  $H$
4. ... know and interpret the standard notation of general solutions to the Harmonic oscillator  $|n\rangle$ , in position, momentum, AND/OR energy basis. (You need not memorize Hermite polynomials or normalization constants)
5. ... use the standard "time evolution" methods from Ch 3 on harmonic oscillator wave functions, and thus find expectation values of  $\langle x \rangle$ ,  $\langle p \rangle$ , or  $\langle H \rangle(t)$

### **McIntyre Ch 10 :**

#### **You should be able to...**

1. ... Compute 1st and 2nd order corrections to energies, and first order corrections to eigenstates, for arbitrary familiar unperturbed Hamiltonians. (See crib sheet for formulas I will give you)
2. ... Be able to make qualitative arguments, e.g. when first order energy corrections will vanish, which "terms" will or will not enter in the sums for 2nd order or wave function corrections
3. ... Be able to solve for first order energy shifts in systems with DEGENERATE states. This means knowing how to identify a degenerate subspace, and then diagonalize the resulting (small, or block diagonal) matrix.
4. ... Be able to interpret the eigenvectors of the degenerate perturbation hamiltonian in terms of "good states" (you should know what that term means!), and find the "good states" for a (small or block diagonal) perturbation matrix

### **McIntyre Ch 11 :**

#### **You should be able to...**

1. ... Describe in your own words the basic physics of the hyperfine interaction, and be familiar with the notation used in the text (**S** for spin of electron, **I** for spin of proton, and **F = I+S** for total spin of the system. You should also be able to sketch and explain and annotate the spectrum of hydrogen's ground state hyperfine levels.

2. ... use "raising and lowering" operators  $L_+$  and  $L_-$  for angular momentum states, including normalization (see crib sheet!)
3. ... Know and use commutation relations for combinations of  $L_+$ ,  $L_-$ , and  $L^2$ .
4. ... Be able to use a relation like  $\mathbf{F} = \mathbf{S} + \mathbf{I}$  to deduce  $\mathbf{S} \cdot \mathbf{I}$  in terms of  $\mathbf{S}^2$ ,  $\mathbf{I}^2$ , and  $\mathbf{F}^2$ , (and generalize e.g. to using  $\mathbf{S} = \mathbf{F} - \mathbf{I}$  to deduce  $\mathbf{F} \cdot \mathbf{I}$ , etc...)
5. ... Also be able to use the relation (in the crib-sheet)  $\mathbf{S} \cdot \mathbf{I} = 1/2 (S_{+I} + S_{-I}) + S_z I_z$  when needed.
6. ... Know the difference between coupled and uncoupled basis states, and be able to write down the coupled "singlet" and "triplet" states of hydrogen's hyperfine ground states in terms of uncoupled states.
7. ... Be able to add (or combine) arbitrary angular momenta, not JUST  $1/2 + 1/2$ . This means being able to use a Clebsch-Gordan table (provided to you) to relate coupled eigenstates to uncoupled ones, or vice-versa

### McIntyre Ch 12 :

#### You should be able to...

1. ... Describe in your own words the basic physics of the relativistic and spin-orbit perturbations
2. ... Use the formula 12.47 (it will be in the crib sheet) for the full fine-structure correction to reproduce/sketch Figure 12.4, including labeling states as McIntyre does, or in Dirac notation
3. ... Describe in your own words the basic physics of the Zeeman effect. (The Hamiltonian is in the crib sheet)
4. ... Know which basis is appropriate for calculating the Zeeman effect for large or small fields, know what "large or small" is in reference to (i.e. referencing the fine-structure splitting!)
5. ... Be able to sketch Zeeman level energy diagrams for small or large fields, including labeling split states in Dirac notation and identifying any remaining degeneracies.

### McIntyre Ch 13 : You should be able to...

1. ... Characterize identical bosons, fermions, or indistinguishable particle states in terms of overall symmetry (or antisymmetry, or neither)
2. ... Use the symmetry/antisymmetry relating spin and spatial parts of the wave function to label /write out the low-lying energy states, and count out their degeneracies, for pairs of non-interacting particles in a given well. You might be asked to present this as a "labeled energy level diagram"
3. ... Set up and interpret problems involving the "exchange interaction" to get different results for the average separation of particles depending on their symmetry
4. ... Qualitatively discuss the Exchange energy when identical particles are interacting (e.g., to interpret Fig 13.6 and explain the meaning of the various terms and splittings)

5. ... Just qualitatively discuss the ordering of filling of atomic shells in the periodic table

### **McIntyre Ch 14 : Time dependent perturbation theory**

#### **You should be able to...**

1. ... Use Eq 14.20 (which will be on the crib sheet) to solve for transition probabilities given a time dependent perturbation
2. ... Make qualitative predictions about the time dependence of transition probabilities and identify which transitions are forbidden (selection rules) based on the  $H'_{fi}$  matrix elements.

### **McIntyre Ch 16 : Quantum Computing**

#### **You should be able to...**

1. ... Form tensor products of states and operators
2. ... Predict "output states" for few- gate configurations of 1-3 qubit systems, with gates including 1, X, Z, H, and controlled-versions of any of these three.
3. ... Predict probabilities of single particle measurements, given output states. Also predict "conditional" probabilities, of the form "suppose particle #1 is measured and yields 0, what is then the probability that #2 will be 1" etc.
4. ... Distinguish entangled from not-entangled states, and use Bell-state notation (basis states will be given in the crib sheet)