In a prior tutorial, some of you intuitively figured out the formula for momentum. So, the formula *p* = *mv* connects closely to common sense. Is momentum unusual in that regard, or should you expect physics equations to make sense in general? We’ll use a new concept, *torque*, to explore this issue.

# Intuitions about balancing

A ruler is balanced on a pivot, as shown here. If we hang weights from the ruler, under what conditions will the system stay balanced?

## A washerclip (washer attached to a paper-clip with one “leg” unfolded) is hung 20 cm to the left of the pivot. *Intuitively*, where should you hang a *pair* of washerclips to keep the system balanced?

20 cm

?

### Make a prediction and discuss it with your group.

### *If you want*, test your prediction. But if you’re confident, skip the experiment. **In this tutorial, just do the experiments you want.** The point is to learn stuff, not to go through procedures.

## The washerclip now hangs 15 cm to the left of the pivot. A student will hang weights 5 cm to the right of the pivot. Intuitively, how many washerclips should she use, to keep things balanced? Predict, discuss.

15 cm

5 cm

?

## It’s time to “translate” your reasoning into an equation. Let *W*1 denote the weight of the object placed to the left of the pivot. Let *d*1 denote its distance from the pivot. Similarly, *W*2 and *d*2 refer to the second object. Generalizing from parts A and B, write an equation relating *W*1, *d*1, *W*2, and *d*2, an equation that’s satisfied when the system is balanced.

*W*1

*W*2

*d*1

*d*2

## Let’s try some specific scenarios.

### How can you make six washerclips balance three washerclips?

Is there only one way to make them balance?

### A 20-ounce weight is placed 17 cm to the left of the pivot. Where should you place a 33-ounce weight to balance the system?

✯ *Consult an instructor before you proceed.*

# From door-pushing intuitions to *torque*

Now you’ll think about something totally different from balancing meter sticks. By the end of tutorial, though, you’ll see how it all ties together.

As you know a *force* is the effectiveness of a push at making something change its motion. Similarly, a *torque* is the effectiveness of a push at making something change its rotational motion. For instance, you must apply a force to get a cart moving. Similarly, a helicopter’s engine must apply a torque to the rotor to get it rotating.

To figure out the formula for torque—and please “play along” even if you know it—you’ll think about pushing a door to make it rotate (swing).

## Imagine making a heavy door swing open by giving it a brief, hard push. First, you push at its center, “push 1” in the diagram. Then, you push near the edge, “push 2.” Both pushes are equally hard and last the same time.

push 2

push 1

TOP-DOWN VIEW

(Top of door as seen from ceiling)

### (*Prediction*) Which push is more effective at making the door swing open? In other words, which push generates a bigger torque? Why?

### (*Experiment*) To test your prediction, or just to get a hands-on feel, you can do the experiment.

### Suppose push 2 is applied exactly twice as far from the hinge as push 1. *Intuitively*, how does the torque generated by push 2 relate to the torque generated by push 1? Is it half as big? Twice as big? Four times as big?

## Again, let’s compare the effectiveness of two pushes. This time, both pushes are applied to the same spot on the door. But push 4 is stronger than push 3.

push 4

push 3

### Easy question: Which push, 3 or 4, is more effective at making the door swing open? In other words, which push generates a bigger torque?

### Suppose the force exerted by push 4 is exactly twice as strong as the force exerted by push 3. Intuitively, how does the torque generated by push 4 relate to the torque generated by push 3? Is it half as big? Twice as big? Four times as big?

## Building on your above answers, figure out a formula for torque in terms of *F* and *r*, where *F* is the force exerted by a push, and *r* is the distance from the pivot to the spot where the push is applied. *Explain how the formula expresses your intuitions from parts A and B above*. (Physicists use the Greek letter ** for torque.)

✯ *Consult an instructor before you proceed.*

# Applying torque to the meter stick

Now you’ll tie together the two seemingly-unrelated scenarios you’ve addressed in this tutorial.

## A weight *W*1 is placed a distance *d*1 to the left of the pivot, as shown here. The weight exerts a torque on the meter stick, making it swing (rotate) counterclockwise. In terms of *W*1 and *d*1, what torque does that weight generate on the ruler? (Use the formula you just figured out for torque!)

*W*1

*d*1

## Now a weight *W*2 is placed a distance *d*2 to the right of the pivot, as shown here, exerting a clockwise torque on the meter stick. In terms of *W*2 and *d*2, what torque does that weight generate on the ruler?

*W*2

*d*2

## Now both weights are placed on the meter stick. To keep the system in balance, how must the counterclockwise torque generated by weight 1 relate to the clockwise torque generated by weight 2? Specifically, to make things balance, should **1 be greater than, less than, or equal to **2? Why?

*W*1

*W*2

*d*1

*d*2

## Write an equation, in terms of *W*1, *d*1, *W*2, and *d*2, expressing the insight that the torques are balanced.

## The equation you wrote in part D probably looks a lot like the equation you wrote back on the first page of the tutorial. What is the difference in how you generated them? What is the point of doing it both ways? Did your common-sense ideas contribute to zero, one, or both ways of coming up with that equation?

# Balancing multiple objects

For this section, we’ll return to experimenting with the balanced ruler and washers.

## Can you balance two washerclips at the same location on one side with two washerclips at different locations on the other side? How?

## How many paperclips weigh the same as one “naked” washer (with no clip attached)? The challenge here is to be exact: something like “the washer weighs the same as 10.3 paperclips” rather than “about 10 paperclips” or “between 10 and 11 paperclips.”

## Predict whether the following arrangements of washers will balance. (After predicting, you can set up each arrangement to check your predictions, if you wish.)

## In the arrangements below, where could you add a single washer to make the ruler balance?

### 

## Do arrangements that balance always have the same mass on each side of the pivot? When do they, and when don’t they?

# Balancing extended objects

## Unscrew the clamp on the ruler and push it off-center a little bit, then tighten the clamp again.

### Use washerclips to balance the off-center ruler on the pivot.

### Explain what’s going on in terms of torque. How many forces are you thinking of, and where are they acting? Draw a diagram to show what you mean.

### Use your understanding of torque to determine the mass of the ruler. (Express the mass in terms of the washerclip units you have been using so far.) Explain your reasoning.

## Imagine balancing a hammer on your finger (horizontally).

### Again, explain what’s going on in terms of torque. How many forces are you thinking of, and where are they acting? Draw a diagram.

### How does the mass of the part of the hammer on the left side of your finger compare to the mass of the part of the hammer on the right side of your finger? Explain how you know.