**Test A for Larmor Precession**

*All of the following questions refer to this system:*

An electron is in an external magnetic field $B$ which is pointing in the $z$ direction. The Hamiltonian for the electron spin is given by $\hat{H} = -\gamma B \hat{S}_z$ where $\gamma$ is the gyromagnetic ratio and $\hat{S}_z$ is the $z$ component of the spin angular momentum operator.

**Notation:** $\hat{S}_z |\uparrow\rangle_z = \frac{\hbar}{2} |\uparrow\rangle_z$, and $\hat{S}_z |\downarrow\rangle_z = -\frac{\hbar}{2} |\downarrow\rangle_z$.

*For reference, the eigenstates of $\hat{S}_x$ and $\hat{S}_y$ are given by:*

$|\uparrow\rangle_x = \frac{1}{\sqrt{2}} (|\uparrow\rangle_z + i|\downarrow\rangle_z)$, $|\downarrow\rangle_x = \frac{1}{\sqrt{2}} (|\uparrow\rangle_z - i|\downarrow\rangle_z)$

$|\uparrow\rangle_y = \frac{1}{\sqrt{2}} (|\uparrow\rangle_z + |\downarrow\rangle_z)$, $|\downarrow\rangle_y = \frac{1}{\sqrt{2}} (|\uparrow\rangle_z - |\downarrow\rangle_z)$

1. If the electron is initially in an eigenstate of $\hat{S}_x$, does the expectation value of $\hat{S}_x$ depend on time? Justify your answer.

2. If the electron is initially in an eigenstate of $\hat{S}_x$, does the expectation value of $\hat{S}_y$ depend on time? Justify your answer.

3. If the electron is initially in an eigenstate of $\hat{S}_x$, does the expectation value of $\hat{S}_z$ depend on time? Justify your answer.
4. Consider the following statements from Andy and Caroline when the electron is initially in an eigenstate of \( \hat{S}_x \) (the x component of the spin angular momentum):

**Andy:** The electron will NOT be in an eigenstate of \( \hat{S}_x \) forever because the state will evolve in time.

**Caroline:** I disagree. If a system is in an eigenstate of an operator corresponding to a physical observable, it stays in that state forever unless a perturbation is applied. With whom do you agree? Explain.
A. Andy
B. Caroline

5. If the electron is initially in an eigenstate of \( \hat{S}_z \), does the expectation value of \( \hat{S}_x \) depend on time? Justify your answer.

6. If the electron is initially in an eigenstate of \( \hat{S}_z \), is there any precession of \( \hat{S}_x \) about the z axis? If your answer is yes, explain why and give an example of a situation where there will be no precession of \( \hat{S}_x \) about the z axis. If your answer is that there is no precession for the given case, explain why.