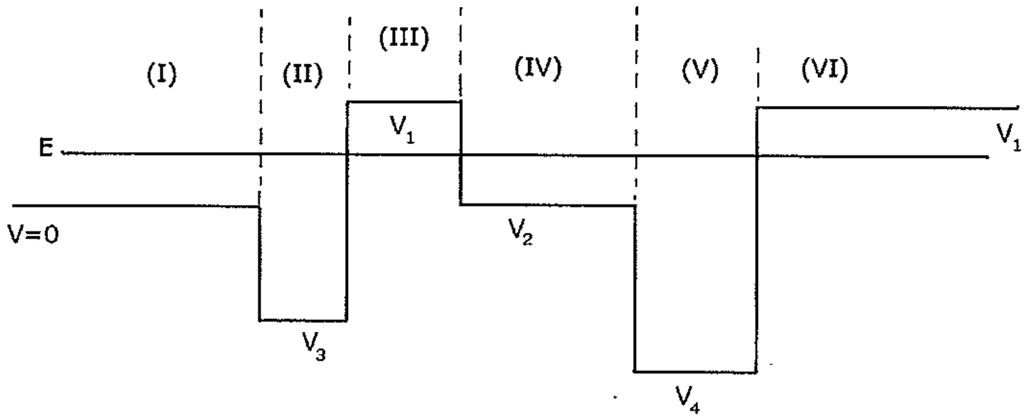
22 pts Solution

Test A for Bound & Scattering States

1. A particle with energy E interacts with a 1-D potential energy function $V(x)$ as shown below. Answer the following questions.



(a) For a classical particle, if at time t=0 the particle is found in region (II), write down all the possible regions (separated by the dash lines) where you may find the particle at time t>0. Explain.

1 pt

I, II

particle does not have enough energy to reach region (III)

(b) For a quantum mechanical particle, write down all the regions where you may find the particle with energy E shown. Is the particle in a bound or scattering state? Explain.

2 pts

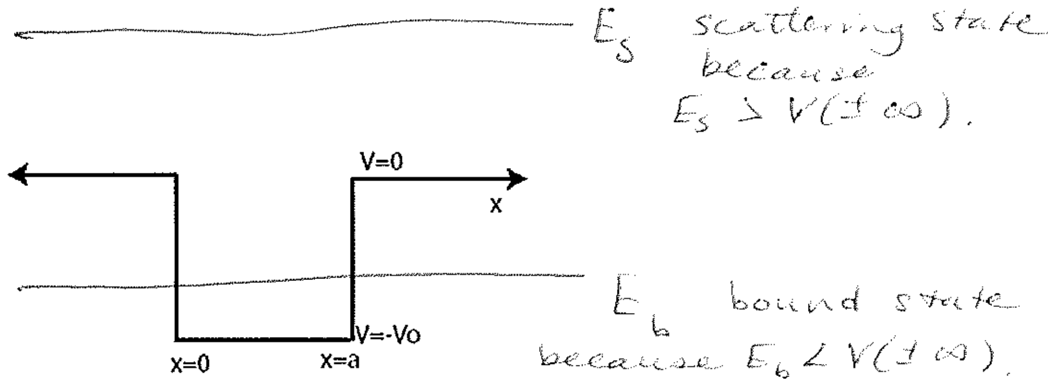
I, II, III, IV, V, VI

particle can tunnel to classically forbidden regions.

$E > V(-\infty)$, so the particle is in a scattering state.

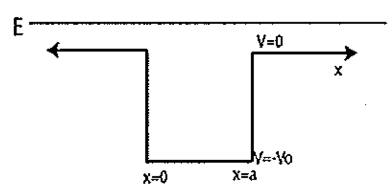
2 pts

2. An electron interacts with a finite square well illustrated below. The electron has an energy E_b if it is in a bound state, and it has an energy E_s if it is in a scattering state. Draw examples of possible energies E_b and E_s on the figure below if it is possible to draw them. Provide your reasoning. If it is not possible to have either bound or scattering states, explain why not.



1 pt

3. Choose all of the following statements that are correct about an electron with energy E interacting with a finite square well, as shown schematically in the figure below. The electron is launched from the left.



(I) The electron has a non-zero probability of bouncing back by the potential energy well.

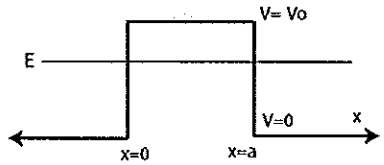
(II) The electron has a non-zero probability of passing through the potential energy well.

(III) It is possible that the electron is trapped by the potential energy well and the wavefunction at $x \to +\infty$ is zero.

A. (I) only
B. (II) only
C. (I) and (II) (circled)
D. (I) and (III)
E. All of the above

1 pt

4. As shown below, a particle with energy $E < V_0$ is interacting with a potential energy barrier. Choose all of the following statements that are correct.

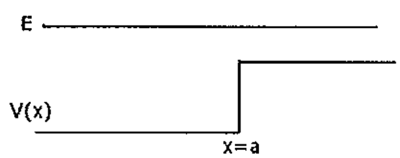


(1) The particle is in a scattering state.
(2) The particle is in a bound state.
(3) The particle is in a bound state between $0 \le x \le a$ and in a scattering state elsewhere.

(A.) 1 only B. 2 only C. 3 only D. none of the above E. not enough information

1 pt

5. A particle with energy E is interacting with a piecewise continuous potential energy $V(x)$ as shown. As can be seen from the Figure, $(E - V(x)) > 0$ for the region $x > a$. Choose all of the following functions that correctly represent the wavefunction in the region $x > a$ for a particle launched from the left. ($k > 0$ for all the options and A, B and C are constants). Ignore the normalization issues of wavefunction.



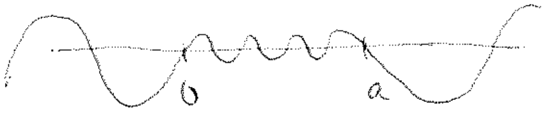
(1) Ae^{ikx} (2) $B\sin(kx) + C\cos(kx)$ (3) Ae^{kx}

A. 1 only B. 2 only C. 3 only (D.) 1 and 2 only E. 2 and 3 only

2 pts

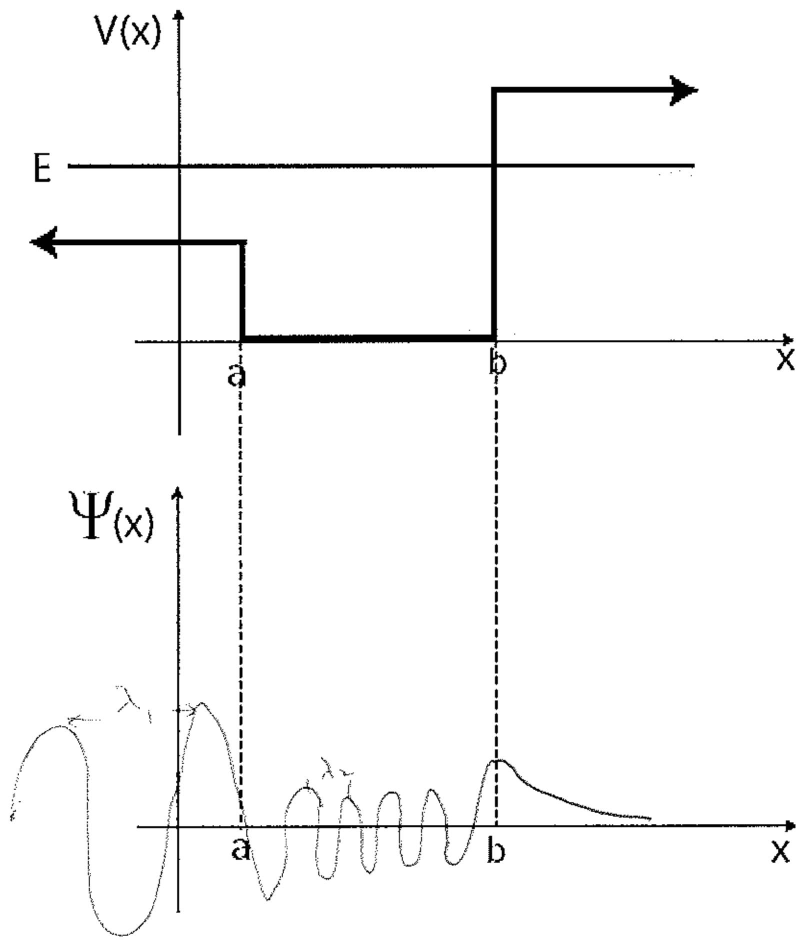
6. Student A says that the scattering state wavefunction for an electron in a finite square well can never be symmetric about the center of the well because the electron has to be launched from either left or right. Student B says that the scattering state wavefunction in a finite square well can be symmetric about the center of the well because the electron need not be launched from left or right. Explain why you agree or disagree with each student. Qualitatively sketch a wavefunction to support your answer.

Agree with student B. Electron does NOT have to be launched from one side, It could be in the well and excited, in which case there is no preference



- Please draw only the real part of the wave function in all the questions.

7. Sketch one possible stationary state wave function for an electron in the following finite square well with energy *E*. Explain your drawing. Explicitly comment on the wavelength in the different regions.



3 pts

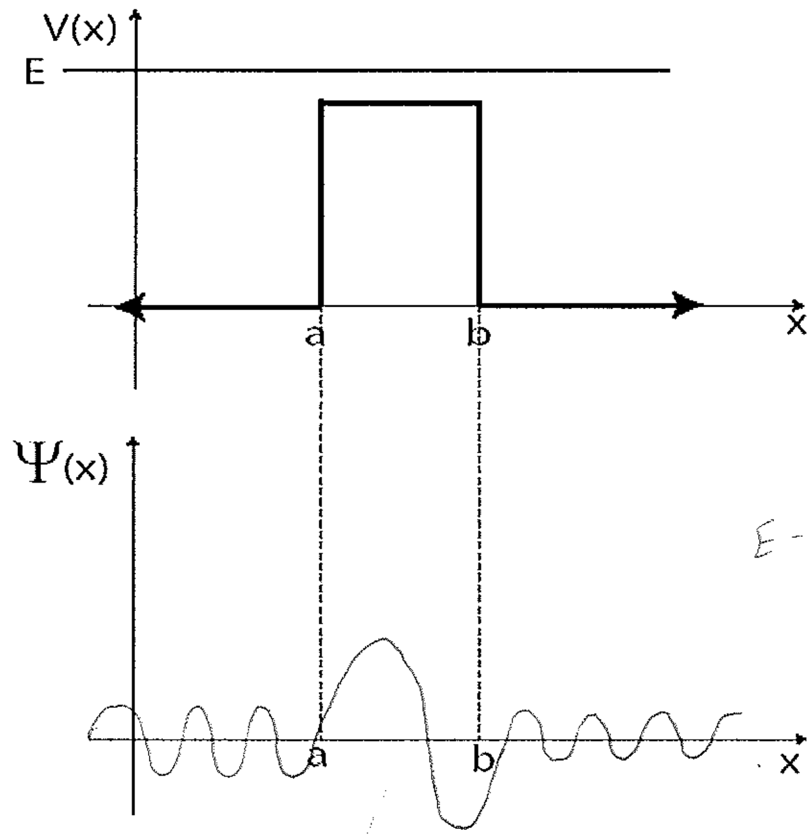
$x < a$ λ_1 larger $\lambda_2 < \lambda_1$

$a < x < b$ λ_2 smaller because p is larger

$x > b$ decays

8. Sketch one possible stationary state wave function for an electron interacting with the following finite piecewise continuous barrier with energy E. Explain your drawing. Explicitly comment on the wavelength in the different regions.

3 pts



$K = E - V$

$K = \frac{p^2}{2m}$

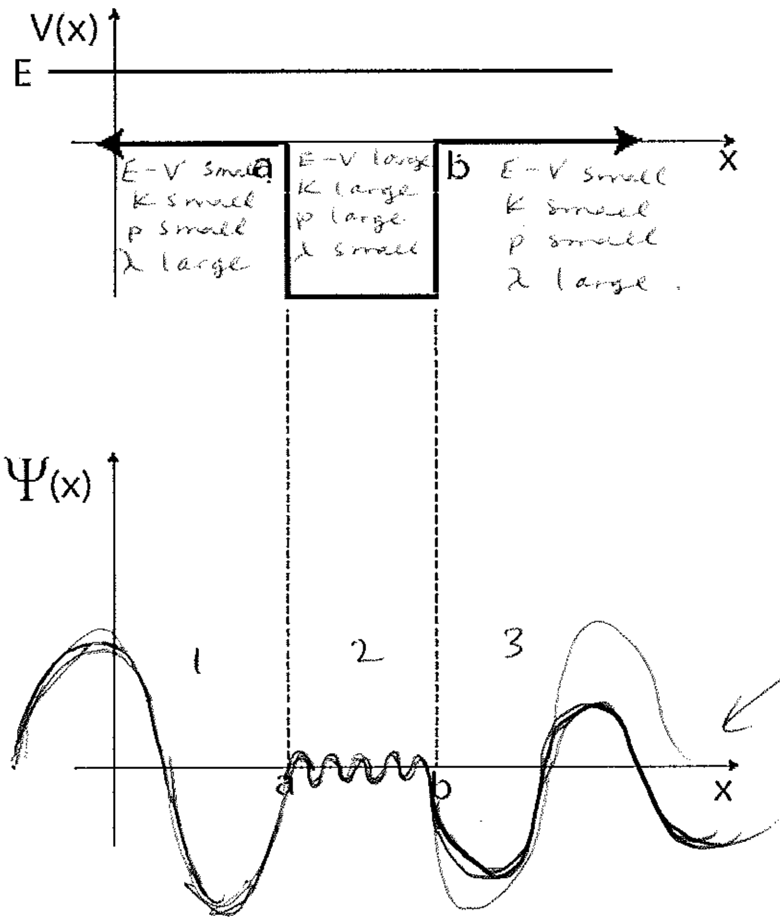
$E - V\uparrow, K\uparrow, p\uparrow, \lambda\downarrow$

$x < a$ smaller λ because p is larger

$a < x < b$ larger λ beause p is smaller there

$x > b$ same as $x < a$, smaller λ because p is smaller there.

9. **Suppose the electron is incident from** $x = -\infty$. Sketch one possible stationary state wave function for the electron interacting with the following finite square well with energy *E*. Explain your drawing. Explicitly comment on the wavelength in the different regions.



3 pts

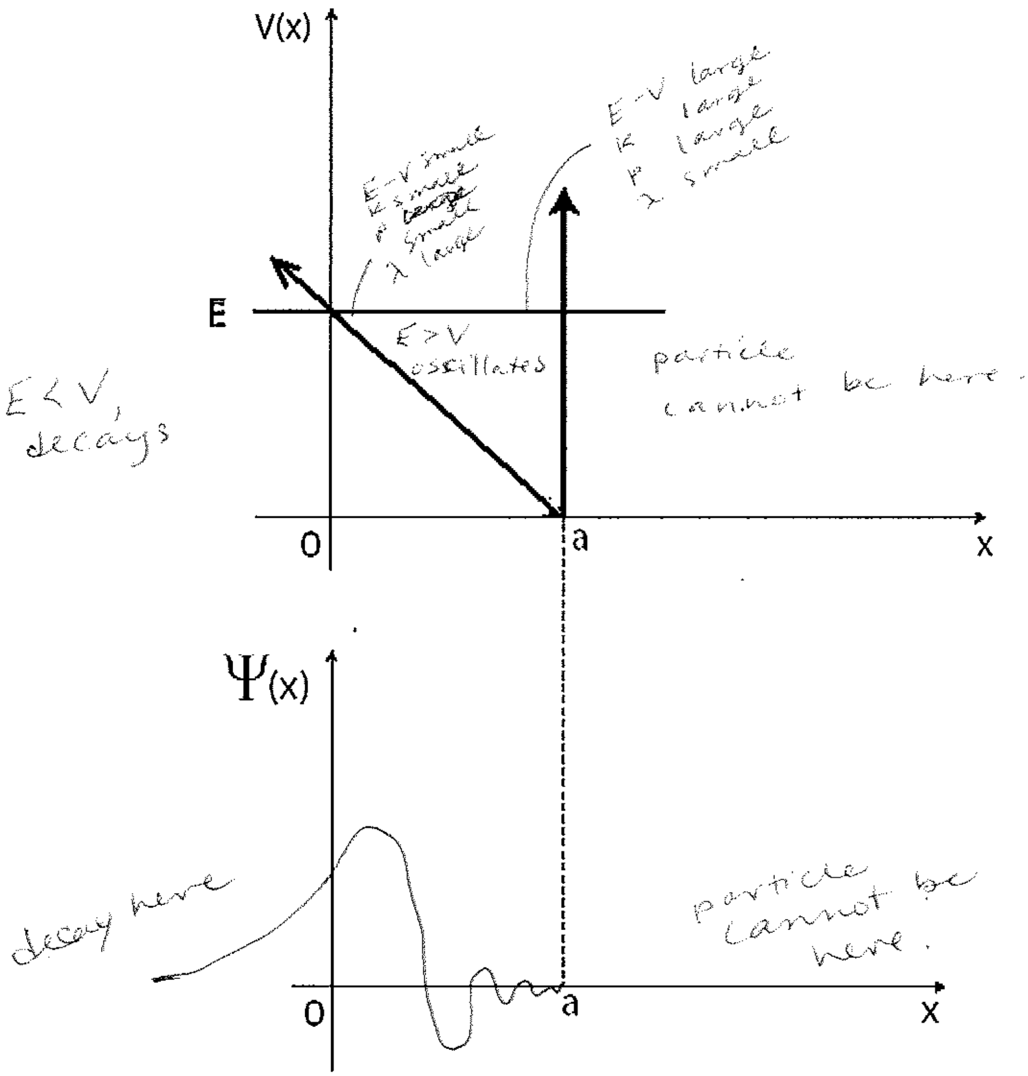
amplitude smaller here because it was launched from left,

$a < x < b$ λ shorter because p is larger there.

$\lambda_1 = \lambda_3 > \lambda_2$

10. Sketch one possible wave function for an electron with energy E interacting with the triangular potential energy well $V(x)$ as shown ($V(x=0)=E$ as shown). The potential energy is infinite in the region x>a. Explain your drawing. Explicitly comment on the wavelength in the different regions.

3 pts



between 0 + a, wavelength gradually gets smaller because p gets larger!