Test A for measurement

All of the questions below refer to an isolated system in which a particle is in a 1-D infinite square well with Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$ $(V(x) = 0$ *when* $0 < x < a$, $V(x) = +\infty$ *otherwise). The energy eigenvalues are* $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ *(n=1,2,3,…), and the energy eigenstate corresponding to* E_n *is* $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$ *when* $0 < x < a$ *and* $\psi_n(x) = 0$ *elsewhere.*

1. Circle all of the following statements that are correct if the wave function is $\frac{\psi_1(x)+\psi_2(x)}{\sqrt{2}}$:

(I) The **position** of the electron is well defined. (Well-defined value of an observable in a given state implies that measurement of that observable will yield a particular value with 100% probability.)

(II) The **momentum** of the electron is well defined.

(III) The **energy** of the electron is well defined.

2. An electron is in the state given by $\frac{\psi_1(x)+\psi_2(x)}{\sqrt{2}}$. Which one of the following outcomes could you obtain if you measure the energy of the electron?

A. $E_1 + E_2$

B. $(E_1 + E_2)/2$

C. Either E_1 or E_2

D. Any of the E_n *(n=1,2,3,…)*

E. Any value between E_1 and E_2

3. An electron is in the first excited state $\psi_2(x)$. If you measure the position of the electron, where can you expect to find the particle with the highest probability? Draw a rough sketch of the electron's wave function showing what it may look like right AFTER the position measurement.

4. The state of an electron at t=0 is given by $\psi(x) = Ax$ when $0 < x < \frac{a}{2}$, $\psi(x) = A(a-x)$ when $\frac{a}{2} \le x < a$ and $\psi(x) = 0$ elsewhere. Here A is the normalization constant. What is the probability that an energy measurement at time t=0 yields E_2? (Ignore the fact that the first derivative of the wavefunction is not continuous. If there is an integral in your expression for the probability, you need not evaluate the integral but set it up properly with appropriate limits.)

5. If you make a measurement of position on an electron in the ground state and wait for a long time before making a second measurement of position, do you expect the outcome to be the same in the two measurements? Explain.

6. If you make a measurement of energy on an electron in the ground state and wait for a long time before making a second measurement of energy, do you expect the outcome to be the same for the two measurements? Explain.