Test B for measurement

1. The energy eigenvalues for a one dimensional simple harmonic oscillator (SHO) are $E_n = (n + \frac{1}{2})\hbar\omega$. You perform a measurement of the energy of a particle interacting with a SHO potential energy well and obtain $\frac{3}{2}\hbar\omega$. Circle all of the following statements that are correct about the wave function after the measurement.

(I) The wave function will become localized about a certain value of position.

(II) The wave function will become a stationary state wave function right after the energy measurement.

(III) The wave function will continue to be a stationary state wave function long after the energy measurement.

2. Circle all of the following statements that are correct:

(I) The stationary states refer to the eigenstates of any operator corresponding to a physical observable.

(II) If a system is in an eigenstate of any operator that corresponds to a physical observable, it stays in that state unless an external perturbation is applied.

(III) If a system is in an energy eigenstate at time t=0, it stays in the energy eigenstate unless an external perturbation is applied.

All of the questions below refer to a system in which a particle is in a 1-D infinite square well with Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$ $(V(x) = 0$ *when* $0 < x < a$, $V(x) = +\infty$ *otherwise). The energy eigenvalues are* $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ *(n=1,2,3,…), and the energy eigenstate corresponding to* E_n *is* $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$ *when* $0 < x < a$ *and* $\psi_n(x) = 0$ *elsewhere.*

The state of an electron in a one dimensional infinite square well of width *a* at *t*=0 is given by $\psi(x,0) = \sqrt{2/7}\psi_1(x) + \sqrt{5/7}\psi_2(x)$ where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and the first excited state of the system. Answer the following questions (3~6).

3. If you measure the energy of the electron after a time t, what possible energies could you obtain and what is the probability of each? Explain.

4. If the energy measurement yields $4\pi^2\hbar^2/(2ma^2)$, what is the wave function of the system right after measurement? Make a qualitative sketch of the wave function right after the measurement of energy.

5. Immediately after the energy measurement in question 4, you measure the position of the electron. What possible values could you obtain, and what is the corresponding probability density? Explain. Is there any position inside the well that can not be observed in this case? Explain.

6. After the position measurement in question 5, suppose you wait for a time t before measuring the position again. Would the possible values of position and the probability of measuring each be different from question 5? Explain.