Reflective Homework

(1) Consider the following statement: "A magnetic field applied in the z direction cannot change the probability of measuring the different values of S_x (x component of spin angular momentum) because the magnetic field is orthogonal to the eigenstates of $\hat{S}_x$". Clearly explain why you agree or disagree with the statement.

Notation: $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the orthonormal eigenstates of $\hat{S}_z$ (the z component of the spin angular momentum) of the electron.

(2) $\sqrt{3/7}|\uparrow\rangle + \sqrt{4/7}|\downarrow\rangle$ represents the state of the electron for the spin degree of freedom when you first measure S_z and then S_x in immediate succession. What are the possible values of S_x you will measure, and what are their respective probabilities? Justify your answer.

(3) Suppose after the measurement of S_x you measure S_z in immediate succession. What are the possible values of S_z you will measure, and what are their respective probabilities? Justify your answer.

(4) Consider the following statements:
Statement 1: "An electron spin cannot be in an eigenstate of the spin angular momentum operator $\vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$. In other words, you cannot measure the total spin angular momentum vector $\vec{S}$ of an electron".
Statement 2: "I agree but we can measure the magnitude of the spin angular momentum vector".
Explain why you agree or disagree with each statement.

(5) If you make measurements of "the distance of the electron from the nucleus" on an ensemble of hydrogen atoms all in the ground state, do you expect to find all electrons at the same distance from the nucleus? Justify your answer.

(6) Suppose the state of the system for a one-dimensional infinite square well ($0 \le x \le a$) is an equal linear superposition of the ground state and first excited state as follows: $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ at time $t = 0$. What is the probability of measuring position between x and $x + dx$ at time $t = 0$? What are the probabilities of measuring different values of energy at time $t = 0$? What is the probability of measuring position between x and $x + dx$ at time $t > 0$? What are the probabilities of measuring different values of energy at time $t > 0$? Write all of these probabilities both using the Dirac notation and in the position space.