

Dirac Notation Quiz

Note: For all of the following questions,

- Ignore the normalization issues of the position and momentum eigenstates.
- Assume that a generic Hermitian operator \hat{Q} corresponding to an observable Q only depends on position and momentum operators (\hat{x} and \hat{p} , respectively).
- Assume that $|\Psi\rangle$ denotes a generic state of a system with Hamiltonian \hat{H} .
- Assume that the Hilbert space is infinite dimensional.
- Assume that the particle is confined in a one dimensional physical space.

- Write a momentum eigenfunction with eigenvalue p' in the momentum representation.
 - Write a momentum eigenfunction with eigenvalue p' in the position representation.
- You are given a generic state vector $|\Psi\rangle$. How would you obtain the position space wave function from $|\Psi\rangle$?
- $|x'\rangle$ is a position eigenstate with the eigenvalue x' . $|p'\rangle$ is a momentum eigenstate with the eigenvalue p' . Using the information given, fill the blanks on the right hand side in the following equations. If the left hand side shows an expression in Dirac notation, write it in position or momentum representation on the right hand side and vice versa. There may be more than one correct answer for a given question. Your answer must be something different than what is shown on the left hand side to obtain credit for a given question.
 - $\langle x|x'\rangle =$ _____
 - $\langle x|\hat{p}|p'\rangle =$ _____
 - $\langle p|\hat{p}|p'\rangle =$ _____
 - $x\delta(x - x') =$ _____
 - $\langle x|p'\rangle =$ _____
 - $\langle p|p'\rangle =$ _____
 - $\langle p|x'\rangle =$ _____
 - $\langle x|\hat{Q}|\Psi\rangle =$ _____

4. Show that the wave function in the position representation is the Fourier transform of the momentum space wave function. Show all your work.

5. $|\Psi\rangle$ is a generic state of a quantum system. The states $\{|q_n\rangle, n = 1, 2, 3 \dots \infty\}$ are eigenstates of an operator \hat{Q} corresponding to a physical observable with discrete eigenvalues q_n .

a) Find the expectation value of Q for state $|\Psi\rangle$ using a basis of eigenstates $|q_n\rangle$ and eigenvalues q_n . Show your work.

6. $|\Psi\rangle$ is a generic state of a quantum system. The states $\{|q\rangle\}$ are eigenstates of \hat{Q} with continuous eigenvalues q .

a) What is the probability of measuring observable Q in the interval between q and $q + dq$ in the state $|\Psi\rangle$?

b) Find the expectation value of Q for state $|\Psi\rangle$ using a basis of eigenstates $|q\rangle$ and eigenvalues q . Show your work.

c) Write the spectral decomposition of the identity operator \hat{I} (i.e., the completeness relation), using a complete set of eigenstates $|q\rangle$ of the operator \hat{Q} .