Version B Name KEY

Dirac Notation Quiz

Note: For all of the following questions, ignore the normalization issues of the position and momentum eigenstates. For all questions involving a generic operator $\hat{Q}$, assume that it only depends on position and momentum.

1. You are given a generic state vector $|\Psi\rangle$. How would you obtain the position space wave function from $|\Psi\rangle$?

$$\Psi(x) = \langle x | \Psi \rangle$$

2. a. Write a momentum eigenstate with eigenvalue p' in the momentum representation.

$$\langle p | p' \rangle = \delta(p - p')$$

b. Write a momentum eigenstate with eigenvalue p' in the position representation.

$$\langle x | p' \rangle = \frac{e^{\frac{ip'x}{\hbar}}}{\sqrt{2\pi\hbar}}$$

3. $|x'\rangle$ is a position eigenstate with the eigenvalue x'. $|p'\rangle$ is a momentum eigenstate with the eigenvalue p'. Using the information given, fill the blanks in the following equations with the correct right hand side.

a. $\langle x|x'\rangle =$ $\delta(x-x')$
b. $\langle x|\hat{p}|p'\rangle =$ $p' \, e^{ip'x/\hbar}/\sqrt{2\pi\hbar}$
c. $\langle p|\hat{p}|p'\rangle =$ $p' \, \delta(p-p')$
d. $\hat{x}\delta(x-x') =$ $x'\delta(x-x')$ or $x\,\delta(x-x')$
e. $\langle x|p'\rangle =$ $e^{ip'x/\hbar}/\sqrt{2\pi\hbar}$
f. $\langle p|p'\rangle =$ $\delta(p-p')$
g. $\langle p|x'\rangle =$ $e^{-ipx'/\hbar}/\sqrt{2\pi\hbar}$
h. $\langle x|\hat{Q}|\Psi\rangle =$ $\hat{Q}(\hat{x}, -i\hbar \partial/\partial x)\Psi(x)$

4. Show that the position space wave function is the Fourier transform of the momentum space wave function. Show all your work.

$$\psi(p) = \langle p | \psi \rangle$$

$$\psi(p) = \int_{-\infty}^{\infty} \langle p | x \rangle \langle x | \psi \rangle \, dx$$

$$\psi(p) = \int_{-\infty}^{\infty} \frac{e^{-ipx/\hbar}}{\sqrt{2\pi\hbar}} \psi(x) \, dx$$

fourier transform

All of the questions below refer to an infinite dimensional Hilbert space.

5. $|\Psi\rangle$ is a generic state of a quantum system. $|q_n\rangle$ are eigenstates with discrete eigenvalues q_n ($n = 1,2,3 \ldots$) of an operator $\hat{Q}$ corresponding to a physical observable Q.

a) Find the expectation value of $\hat{Q}$ in the state $|\Psi\rangle$ in terms of eigenstates $|q_n\rangle$ and eigenvalues q_n. Show your work.

$$\langle \psi | \hat{Q} | \psi \rangle =$$

$$= \sum_{n=1}^{\infty} \langle \psi | \hat{Q} | q_n \rangle \langle q_n | \psi \rangle$$

$$= \sum_{n=1}^{\infty} q_n \langle \psi | q_n \rangle \langle q_n | \psi \rangle$$

$$= \sum_{n=1}^{\infty} q_n |\langle q_n | \psi \rangle|^2$$

6. $|\Psi\rangle$ is a generic state of a quantum system. $|q\rangle$ are eigenstates with continuous eigenvalues q of an operator $\hat{Q}$ corresponding to a physical observable Q.

a) What is the probability of measuring observable Q in the interval between q and $q + dq$ in the state $|\Psi\rangle$?

$$|\langle q|\psi\rangle|^2 dq$$

b) Find the expectation value of $\hat{Q}$ in the state $|\Psi\rangle$ in terms of eigenstates $|q\rangle$ and eigenvalues q. Show your work.

$$\langle\psi|\hat{Q}|\psi\rangle =$$

$$= \int_{-\infty}^{\infty} \langle\psi|\hat{Q}|q\rangle\langle q|\psi\rangle\, dq$$

$$= \int_{-\infty}^{\infty} q\, \langle\psi|q\rangle\langle q|\psi\rangle\, dq$$

$$= \int_{-\infty}^{\infty} q\, |\langle q|\psi\rangle|^2\, dq$$

c) Write the completeness relation using a complete set of eigenstates $|q\rangle$ of the operator $\hat{Q}$.

$$\hat{I} = \int_{-\infty}^{\infty} |q\rangle\langle q|\, dq$$