**Post-Test: Dirac Notation Warm-Up**

For all of the following questions:

* Assume that $\left|\left.i\right〉\right., \left.\left|j\right.\right〉, and \left.\left|k\right.\right〉$ form one orthonormal basis and $\left.\left|i'\right.\right〉$, $\left.\left|j'\right.\right〉$, and $\left.\left|k'\right.\right〉$ form another orthonormal basis for the same three dimensional vector space.
* $\left|\left.χ\_{1}\right〉\right.=a\left|\left.i\right〉\right.+b\left.\left|j\right.\right〉+c\left.\left|k\right.\right〉$ and $\left|\left.χ\_{2}\right〉\right.=d\left|\left.i\right〉\right.+e\left.\left|j\right.\right〉+f\left.\left|k\right.\right〉$ are vectors in a three dimensional vector space.
1. For vector $\left|\left.χ\_{1}\right〉\right.$:
	1. Write the components $a, b, and c $in Dirac notation.
	2. Write $\left|\left.χ\_{1}\right〉\right.$ as a column matrix (vector) in the given basis.
2. a. Write the outer product of “ket” vector $\left|\left.χ\_{1}\right〉\right.$ with “bra” vector $\left〈\left.χ\_{2}\right|\right.$.

b. Is this a scalar (number), column vector, row vector, or an operator?

1. Write the identity operator in terms of $\left|\left.i\right〉\right., \left.\left|j\right.\right〉, and \left.\left|k\right.\right〉$ which form a complete set of basis vectors for a three dimensional vector space.
2. Consider the following statement:
* The components of the vector $\left|\left.χ\_{1}\right〉\right. $have fixed values even if we change the basis such that the unit vectors are $\left.\left|i'\right.\right〉$, $\left.\left|j'\right.\right〉$, and $\left.\left|k'\right.\right〉$.

Explain why you agree or disagree with this statement.

1. Choose all of the correct statements about the identity operator (assume three dimensional vector space):
2. In general, given an orthogonal basis $\left|\left.i\right〉\right., \left.\left|j\right.\right〉, and \left.\left|k\right.\right〉$, if we compute the outer product of each unit vector with itself and then add them up, we obtain the identity operator.
3. If we change the basis we have chosen to a different orthogonal basis $\left.\left|i'\right.\right〉$, $\left.\left|j'\right.\right〉$, and $\left.\left|k'\right.\right〉$, if we compute the outer product of each new basis vector with itself then add them up, we will still obtain the identity operator $\hat{I}$.
4. The completeness relation refers to writing the identity operator in terms of a complete set of basis vectors.
5. (I) only
6. (II) only
7. (III) only
8. (I) and (III) only
9. All of the above
10. For the vector $\left|\left.χ\_{1}\right〉\right.,$
	1. Write down the projection operator that projects vector $\left|\left.χ\_{1}\right〉\right.$ along the direction of the unit vector $\left|\left.i\right〉\right.$.
	2. Using the projection operator from 6.a, show what happens to the vector $\left|\left.χ\_{1}\right〉\right.$ when the projection operator acts on it.
	3. Summarize your result in part 6.b in one sentence.
11. Consider the following statement made by Student A about vector $\left|\left.χ\_{1}\right〉\right.$:
* Student A: $\left⟨χ\_{1}\right⟩$ is a vector which points along the z-direction.

Do you agree with Student A? Explain your reasoning.