**Double-slit Experiment with Single Particles: Tutorial warm-up**

Relativistic effects are ignored in this tutorial.

For any constant, e.g., the mass of an electron or muon or Planck's constant, use the following values:

* 1 eV = 1.6 ×10-19 J
* keV = kilo electron volt = 103 eV, meV = milli electron volt = 10-3 eV
* 1 mm = 10-3 m, 1 µm = 10-6 m, 1 nm = 10-9 m, 1 pm = 10-12 m
* Planck's constant = *h* = 6.6 × 10-34 Js
* magnitude of elementary charge (on an electron or proton) = *e* = 1.6 × 10-19 C
* speed of light = *c* = 3 × 108 m/s
* mass of electron = 9.1 × 10-31 kg
* mass of neutron = mass of proton = 1.67 × 10-27 kg
* mass of muon = 1.9 × 10-28 kg
* mass of helium atom = 6.68 × 10-27 kg
* mass of sodium atom = 3.8 × 10-26 kg

In numerical calculations, if you do not see the exact answer as a choice in the multiple-choice questions, choose the nearest value.

**Double slit Experiment with Single Particles**

**Acknowledgment: The simulation used in this tutorial was developed by Klaus Muthsam.**

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**The goals of this tutorial are to review**

* the de Broglie relation which relates the momentum and wavelength of a particle-wave
* the wave nature of particles manifested via the double slit experiment with single particles

Before you begin this tutorial it is strongly recommended that you briefly review diffraction and double slit interference for electromagnetic waves. For example, you can review the single and double slit interference experiment to refresh your memory on the details of what causes the interference pattern to occur.

It is recommended that you work through this tutorial warm-up with a blank sheet of paper and pen so that you can make calculations and write down important ideas you learn as you go along.

**Double-Slit Experiment: Set-up**

Open the simulation of the double-slit experiment. Select English from the language menu and click ok. You will see the experimental set-up for the “double-slit” experiment.

* The red box is the source which can emit different types of particles.
* The black screen has two slits on it. It comes right after the red source box.
* There is a lamp (bulb) in between the double slit and the white screen (We will NOT use this lamp in this tutorial warm-up).
* There is a white screen after the lamp. When particles passing through the double-slit one at a time hit the screen, their interaction with the atoms in the screen causes one red spot on the screen for each particle.

Before using the simulation, answer the following questions.

1. According to the de Broglie relation

(a) *p* = *h*/*λ* where *p* and *λ* are the momentum and the wavelength of the particle wave

(b) *p* = *λ*/*h*

(c) *x* = *h*/*λ* where *x* is the position of the particle-wave

(d) *x* = *λ*/*h*

2. Consider an electron with kinetic energy *K* (assume it is non-relativistic). Can you associate a wavelength with the electron? Select the correct answer.

(a) Yes. Its wavelength is infinite.

(b) No. Electrons are particles and cannot have a wavelength associated with them.

(c) Yes. But it is not possible to find the wavelength from the information about its kinetic energy.

(d) Yes. The wavelength is $λ=h/\sqrt{2mK}$, where *m* is the mass of the electrons and *h* is Planck's constant.

3. Consider a non-relativistic electron with kinetic energy 1 keV. What is its de Broglie wavelength? (You do not need a calculator because an order of magnitude estimate should tell you the correct answer.)

(a) 2 cm

(b) 3 µm

(c) 47 nm

(d) 39 pm

● Now go to the simulation. In the lower right hand corner, you will see a menu box which has control buttons on it. Click on the “source” menu. Once the “source” menu box opens, select electrons from the “kind of particles”. Adjust their energy to the minimum (1 keV). You will see the associated wavelength *λ* which should agree with the answer to the previous question.

4. Consider the following conversation between Mike, Melissa and Arnold:

● Mike: Even if we send one electron at a time from the source, we will observe an interference pattern on the screen after a large number of electrons have hit the screen. Each electron propagates as a wave. While passing through the slits, it is delocalized in space and it can pass through both the slits simultaneously and interfere with itself.

● Melissa: How can sending one electron at a time through the slits produce an interference pattern? An electron can pass through only one slit at a time. If what you are saying is true and the electron is indeed delocalized and can pass through both the slits simultaneously, shouldn’t we see some sort of streak on the screen instead of a single flash?

● Arnold: I agree with Melissa. If the electron is delocalized, how come we only see one narrow flash? Shouldn’t there be more than one flash, or a spread that corresponds to how delocalized each electron is? I don’t think we will see interference in this case. The fact that one flash is produced means that the electron is localized and it can only pass through one slit at a time.

● Mike: An electron is delocalized only BEFORE it interacts with the screen. When an electron interacts with the screen we see a narrow flash, which means that the wave function of the electron has collapsed and the electron has become localized (i.e., its position is known with certainty). However, before interacting with the screen it is delocalized and, while travelling from the source to the screen, it can pass through both slits simultaneously and interfere with itself to produce an interference pattern on the screen.

With whom, if either, do you agree?

(a) Mike

(b) Melissa and Arnold

(c) None of them

Explain.

5. Which one of the following conditions must be satisfied to get bright interference fringes (constructive interference) on the screen? (*n* = 0, ±1, ±2, ±3… is an integer, *d* is the slit distance (slit separation) and the angle $θ$ is defined on the picture below) Hint: Think about the path length difference of the waves from the two slits arriving at a point on the distant screen.

(a) $d\sin(θ)=nλ$

(b) $d\sin(θ)=nλ/2$

(c) $d\sin(θ)=\left(n+\frac{1}{2}\right)λ$

(d) $d\sin(θ)=\left(n+\frac{1}{2}\right)λ/2$

6. Using the diagram on the right, express *y* (the displacement between the central and first bright fringe) in terms of *d* and *D* (the distance from the slits to the screen) when *θ* is small and *d* << D ? (Hint: use the condition for constructive interference and the fact that sin*θ* ≈ tan*θ* for small *θ*.)

(a) *y* = *d*/*D*

(b) *y* = *Dλ*/*d*

(c) *y* = *dλ*/*D*

(d) *y* = *d*

7. Using *D* = 3m, *d* = 400 nm and *λ* = 39 pm, calculate *y* approximately (you do not need a calculator because an order of magnitude estimate is good enough).

(a) 2.9 × 10-4 m

(b) 2.9 × 10-5 m

(c) 2.9 × 10-6 m

(d) 2.9 m

● Now go to the simulation. Go to the source menu and make sure that the (kinetic) energy is adjusted to 1 keV for the electrons. Then, click on the aperture button at the lower right hand corner and adjust the slit width to 100 nm and the slit distance (slit separation) to 400 nm. Note that the value of *D* (slit to screen distance) has been fixed to 3 m and cannot be changed in the simulation.

● Click on the red button just beside the source menu to switch on the source and click on the speed button on the lower right hand menu and press it down for some time before releasing so that the number of hits (third entry from the left hand side on the bottom panel) goes to around 20,000.

● Note: If you hear a voice in German, the memory is full and you have already sent the maximum number of particles allowed by the simulation.

● Click on the screen button below the aperture button. The zoom option in the screen control box changes the length scale on the screen. Look at the width of your screen and compare it with the answer to the preceding question about the fringe separation *y*. Then, decide whether you need a higher or lower zoom to make sure that 2-5 bright fringes appear.

● Note that lower zoom will increase the field of view (length scale of the screen) and make more fringes appear and that as you go down in the zoom menu, the zoom increases. (Note: A zoom of 1000x is optimal for these given parameters.)

● Click on the “theoretical result” button on the screen menu to compare your interference pattern with the theoretical result. Click on the “evaluation button” to see a distribution of particles on the screen (sometimes it is easier to observe dips at certain locations due to destructive interference than to look for destructive interference in vertical bands). Unclick the evaluation button.

8. Using the interference pattern shown by the simulation, estimate the distance between two bright fringes *y* (it is best to measure the distance from the center of one bright fringe to another).

(i) Is your estimation consistent with the value you calculated?

(ii) If the estimation is not consistent, can you reconcile the difference?

9. (i) If you increase the slit distance *d*, what will happen to the distance between two bright fringes?

(a) It will increase.

(b) It will decrease.

(c) It will stay the same.

(d) The interference fringes will vanish.

(ii) Explain your reasoning for your answer in (i).

● Click on the aperture menu on the lower right hand corner. Change the slit distance (slit separation) to 2000 nm keeping the slit width fixed.

● Turn on the source and hold down the speed button for some time to get an interference pattern.

● Adjust the zoom to 10,000x.

● Note that the numbers denoting various zooms have periods instead of commas (as in 10.000x instead of 10,000x) because the German convention for writing numbers is different. Every time you change the zoom, you have to press the speed button to increase the number of hits.

10. Using the interference pattern on the screen, estimate the distance between the fringes.

(i) Did it increase or decrease? Is this result consistent with your response to the preceding question?

(ii) If the result is not consistent can you reconcile the difference?

11. For a muon of 2 meV (milli electron volt) energy, which one of the following is the de Broglie wavelength (no need for a calculator since an order of magnitude calculation will suffice)?

(a) 1.9 × 10-12 m (pm: picometer)

(b) 1.9 × 10-9 m (nm: nanometer)

(c) 1.9 × 10-6 m (µm: micrometer)

(d) 1.9 × 10-3 m (mm: millimeter)

12. For the 2 meV muon, which one of the following is the distance between the fringes *y* (no need for a calculator since an order of magnitude calculation will suffice)? (*y* = *Dλ*/*d*, *D* = 3 m, *d* = 100 µm)

(a) 57 pm

(b) 57 µm

(c) 57 mm

(d) 57 cm

● Go to the source menu on the lower right hand corner. Select myons (muons) from the list. Adjust the energy to 2 meV and compare the corresponding wavelength (next to the energy) with what you calculated earlier.

● Go to the aperture menu on the lower right hand corner to set the slit width to 10 µm and the slit distance to 100 µm. Switch on the source and press the speed button. Keep the speed button pressed for some time to increase the number of hits. Adjust the zoom until you see 2-5 bright fringes on the screen.

● Using the pattern shown on the screen, measure the distance between the fringes *y* and compare it with your answer to the preceding question.

13. (i) What should happen to the distance between adjacent fringes if you increase the kinetic energy of the muons while keeping other parameters the same? (Hint: think about how wavelength is related to kinetic energy and use the equation for *y* you previously found).

(a) It will increase.

(b) It will decrease.

(c) It will stay the same.

(d) The interference fringes will vanish.

(ii) Explain your reasoning for your answer in (i).

● Go to the source menu on the lower right hand corner. Adjust the muon energy to 500 meV. Switch on the source and press the speed button. Adjust the zoom until you see 2-5 bright fringes on the screen.

14. Using the pattern shown on the screen, measure the distance between the fringes.

(a) Did it increase or decrease? Is this consistent with your answer to the previous question?

(b) If it is not consistent, can you reconcile the difference?

15. (i) Instead of 2 meV muons, if you use 2 meV sodium atoms, with the other parameters the same (slit width 10 µm and slit distance 100 µm), what would happen to the distance between two adjacent bright fringes? (Hint: think about how wavelength is related to mass and use the equation for *y* you previously found.)

(a) It will increase.

(b) It will decrease.

(c) It will stay the same.

(d) The interference fringes will vanish.

(ii) Explain your reasoning for your answer in (i).

● Go to the source menu on the lower right hand corner. Select sodium (Na-molecule) from the list and adjust the (kinetic) energy to 2 meV.

● For a given energy, $λ∝ 1/\sqrt{m}$. Since the mass of a sodium atom is higher than that of a muon, its wavelength is less than the wavelength of the muon.

● Go to the aperture menu on the lower right hand corner to set the slit width to 10 µm and the slit distance to 100 µm.

● Switch on the source and press the speed button. Adjust the zoom until you see 2-5 bright fringes on the screen.

16. Using the pattern shown on the screen, estimate the distance between the fringes.

(a) Compared to when you used 2 meV muons, did the distance between the fringes increase or decrease? Is this consistent with your answer to the previous question?

(b) If it is not consistent, can you reconcile the difference?

Summarize your responses so far by completing the table below.

Assume that the distance from the screen to the slits, *D* = 3 m is fixed

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Particle | Mass (*m*) | Kinetic Energy (*K*) | Slit distance *d* | Wavelength λ | *y* = *Dλ*/*d* | *y* observed |
| Electron | 9.1 × 10-31 | 1 keV | 400 nm |  |  |  |
| Electron | 9.1 × 10-31 | 1 keV | 2000 nm |  |  |  |
| Muon | 1.9 × 10-28 | 2 meV | 100 µm |  |  |  |
| Muon | 1.9 × 10-28 | 500 meV | 100 µm |  |  |  |
| Na atom | 3.8 × 10-26 | 1 meV | 100 µm |  |  |  |

17. Consider the following conversation between Mike and Melissa:

● Mike: If instead of electrons or muons, we send photons one at a time through the double slit, there will not be any interference fringes. This is because photons are massless particles and they do not follow the de Broglie relation.

● Melissa: No. There should be an interference pattern. The photons also follow the relation *λ* = *h*/*p* even if it is not called the de Broglie relation in the case of massless particles.

With whom, if either, do you agree?

(a) Mike

(b) Melissa

(c) Neither

Explain your reasoning.

● Go to the simulation and select photon from the source menu. Adjust the energy to 500 eV. From the aperture menu, select any aperture size you wish (however, make sure that the distance between the slits is at least twice the slit width so that diffraction effects are negligible compared to interference effects). Turn on the source. Click on the speed button and press it down for some time before releasing it (adjust the zoom to 100,000x). Do you observe interference fringes? The presence of the fringes should support Melissa's statement.

18. Consider the following conversation between Tom and Eve about gunballs (radius of about 1/10 mm) passing through the slits with the slit width 1 mm and slit distance 25 mm:

● Tom: If we send gunballs through the double slit with these parameters, we should observe interference fringes but with a somewhat reduced fringe separation.

● Eve: I don’t think so. Gunballs are classical particles and have a large mass. Therefore, their momentum will be relatively large and the de Broglie wavelength will be essentially zero (immeasurably small). Each gunball will pass through one of the slits, and there should not be an interference pattern but two lines on the screen in front of the two slits.

With whom, if either, do you agree?

(a) Tom

(b) Eve

(c) Neither

Explain your reasoning.

● Go to the simulation and select gunballs from the source menu. From the aperture menu, select the slit width to 1 mm and slit distance to 25 mm. Keep zoom at 1x. Turn on the source. Click on the speed button and hold it down for some time before releasing it. Do you observe the interference fringes? (Hint: If interference fringes are formed, the central region on the screen should have constructive interference).

Change the slit distance to 10 mm and repeat the simulation. Do you see the interference fringes?

The absence of the fringes should support Eve's statement.

● Now go to the source menu and select proton. Adjust the energy to 500 µeV. From the aperture menu, set the slit width to 1 µm and slit distance to 10 µm. Calculate the distance *y* between two bright fringes for the given parameters and compare it with what you observe on the screen.

19. If you use neutrons instead of protons of the same energy 500 µeV (keeping the slit distance fixed), which one of the following statements is true?

(a) The distance between the fringes should increase significantly because neutrons do not have a charge.

(b) The distance between the fringes should decrease significantly because neutrons do not have a charge.

(c) The interference pattern will not change significantly because both neutrons and protons have approximately the same mass and identical energy.

(d) The interference pattern will not change significantly because both neutrons and protons are made of quarks.

● Now go to the source menu and select neutron. Keep the energy to 500 µeV and the zoom at 1000x.

Do you observe any significant changes in the interference pattern? Compare the fringe separation for the neutron with what you wrote down for the proton in the last observation.

**Summary**

● Microscopic “particles”, e.g., electrons, protons, muons, sodium atoms etc. can behave as waves in certain experiments, and as particles in other experiments. This is what is referred to as “the wave-matter duality”.

● The de Broglie relation, *λ* = *h*/*p* where *p* is the momentum of the particle is used to obtain the wavelength associated with a “particle”.

● One interesting aspect of the double-slit experiment in particular is that in this experiment, the “particles” used behave both as waves and as particles.

● While travelling from the particle source to the screen, each “particle” behaves as a wave. It is delocalized in space and can go through both slits simultaneously (and interfere with itself).

● The interaction with the screen collapses the wave function of the “particle” which becomes localized (and therefore behaves as a particle).

● Macroscopic “particles”, even if they are small (sand particles for example of radius of a fraction of a mm) do not exhibit interference in the double slit experiment because their associated wavelength is essentially zero (immeasurably small – e.g., any length scale smaller than 10-20 m).

● Interference phenomena have been observed for large molecules (comprised of more than several hundred atoms, see for example S. Eidenberger et. al., “Matter-wave interference with particles selected from a molecular library with masses exceeding 10,000 amu” – available at <http://pubs.rsc.org/en/content/articlehtml/2013/cp/c3cp51500a>).

● The mass of these molecules is about 18 orders of magnitude less than for example, the mass of the finest grain of sand.

● The associated de Broglie wavelength of these molecules is on the order of 10-13 m compared to 10-30 for a very fine, slow moving grain of sand.

● For a non-relativistic particle with kinetic energy *K*, the momentum is given by $p=\sqrt{2mK}$.

● Constructive interference occurs when *d* sin*θ* = *nλ*.

● For small *λ*, the distance between the fringes *y* = *Dλ*/*d*, where *d* is the distance between the slits and *D* is the distance between the screen and the slits.

● Substituting $λ=h/\sqrt{2mE}$ into *y* = *Dλ*/*d* we note that $y∝1/\sqrt{m}$ for fixed *E*, *d* and *D* or $y∝1/\sqrt{E}$ for fixed *m*, *d* and *D* or $y∝1/d$ for fixed *E*, *m* and *D*.

● Since *y* = *Dλ*/*d*, if both the mass *m* and the energy *E* of the particle change such that *λ* is held fixed, *y* will be unchanged.