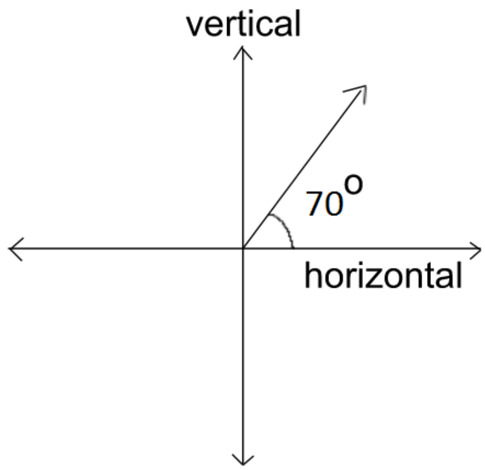
**Pretest**

All of the angles (e.g., $70^0$ shown below) are measured with respect to the horizontal polarization axis toward the vertical polarization axis in the counterclockwise fashion. Assume that polarizers do not reflect any light (they either absorb light or let it pass through).




Alice sends single photons randomly either with $+70^0$ polarization or $0^0$ polarization (horizontal polarization) with equal probability to Bob. Bob uses a polarizer with polarization axis oriented either at $-20^0$ or $90^0$ (vertical) randomly with equal probability. Behind his polarizer is a 100% efficient photodetector which detects every photon that passes through the polarizer (assume that the only source of photons is the one that Alice is using). Alice and Bob each know the protocol for sending single photons (i.e., Bob knows that Alice is sending single photons randomly either with $+70^0$ polarization or $0^0$ polarization with equal probability to him). Alice and Bob conduct a very large number of such experiments.

Use the above protocol to answer the following questions:

1. Bob uses the polarizer with a $-20^0$ polarization and his photo detector does not click. Choose all of the following statements that can be inferred based upon the above protocol used by them:

   (I) Bob is 100% sure about the polarization of the photon sent by Alice.

   (II) Alice must have sent a photon with a $+70^0$ polarization.

   (III) Alice must have sent a photon with a $+0^0$ polarization.

   **(a)** (I) only
   **(b)** (II) only
   **(c)** (III) only
   **(d)** None of the above

2. Bob uses a polarizer with a $-20^0$ polarization and his photo detector clicks. Choose all of the following statements that can be inferred based upon the above protocol used by Alice and Bob:

   (I) Bob is 100% sure about the polarization of the photon sent by Alice.

   (II) Alice must have sent photon with a $+70^0$ polarization.

   (III) Alice must have sent photon with a $+0^0$ polarization.

   **(a)** (II) only
   **(b)** (III) only
   **(c)** (I) and (II) only
   **(d)** (I) and (III) only

3. Suppose Alice transmits a photon with her polarizer set to $70^0$. Bob uses a $90^0$ polarizer to intercept it. Write down the probability that the photon will pass through Bob's polarizer.

4. Alice transmits a photon with $0^0$ polarization and Bob uses a $-20^0$ polarizer. Which one of the following statements is true?
   (a) The photon is blocked by his polarizer with a 100% certainty.
   (b) Approximately 88.3% of the photons will pass through the polarizer.
   (c) Approximately 11.7% of the photons will pass through the polarizer.
   (d) The photon will pass through with a 100% certainty.

5. Bob uses a $90^0$ polarizer and the detector does not click. Can he infer the polarization state of the photon that Alice sent? If so, what is it?
   (a) Yes. Alice must have sent a photon with $0^0$ polarization.
   (b) Yes. Alice must have sent a photon with $70^0$ polarization.
   (c) No. Alice could have sent a photon with either polarization ($0^0$ or $70^0$).
   (d) None of the above.

6. Choose all of the following statements that are correct based upon the protocol described:
   (I) Whenever Bob's detector clicks, he can infer the polarization of the photon that Alice sent.
   (II) Whenever Bob's detector does not click, he cannot infer the polarization of the photon that Alice sent.
   (III) If Alice sends a photon with $0^0$ polarization and Bob uses a $-20^0$ polarizer, that photon will be partly absorbed and partly transmitted.
   **(a)** (I) only
   **(b)** (II) only
   **(c)** (I) and (II) only
   **(d)** (I) and (III) only

7. Complete the third column of the following table by recording "the probability that a photon will pass through and hence Bob's detector clicks":

| | | Probability of detector clicking |
|---|---|---|
| Alice | Bob uses $-20^0$ polarizer | |
| transmits $70^0$ | Bob uses $90^0$ polarizer | |
| Alice | Bob uses $-20^0$ polarizer | |
| transmits $0^0$ | Bob uses $90^0$ polarizer | |

8. Using the table above for the case described in the preceding question, calculate the percentage of measurements in which Bob is 100% sure about the polarization of the photon that Alice sent out of all of the experiments that Alice and Bob conduct.

## Basics

1. "Bit" is an abbreviation for which one of the following?
   (a) Binary digit
   (b) Best kit
   (c) Binary kit
   (d) None of the above

2. Choose all of the following that can be used as a bit:
   (I) a measuring device
   (II) powers of ten
   (III) something that can be in one of two distinct states
   (a) (I) only
   (b) (II) only
   (c) (III) only
   (d) None of the above.

3. Choose all of the following statements that are true:
   (I) A bit is the smallest unit of information in an information processor.
   (II) A single bit can hold only one of two values, which can be labeled as 0 and 1.
   (III) Any information can be represented by combining bits into larger aggregates.
   (a) (I) and (II) only
   (b) (I) and (III) only
   (c) (II) and (III) only
   (d) (I), (II) and (III)

4. Choose all of the following that can be used as a bit:

   (I) the left and right orientations of a magnetic domain

   (II) the voltage across the plates of a capacitor with charge above and below a predetermined cut-off value

   (III) head and tail of a coin

   (a) (I) and (II) only
   (b) (I) and (III) only
   (c) (II) and (III) only
   (d) (I), (II) and (III)

5. For a two-state system in quantum mechanics (e.g., spin one-half or polarization states of a photon), what is the dimensionality of the associated vector space?

   (a) 2 dimensional
   (b) 4 dimensional
   (c) 6 dimensional
   (d) 8 dimensional

6. How many linearly independent vectors (basis vectors) do you need to represent any vector in a two dimensional vector space?

   (a) 2
   (b) 4
   (c) 6
   (d) 8

7. The quantum-mechanical analogue of a classical bit is called a "qubit". A qubit can be described as $|q\rangle = \alpha|0\rangle + \beta|1\rangle$ where $|0\rangle$ and $|1\rangle$ are two orthogonal states of a quantum system. Also, $\alpha$ and $\beta$ are two complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. Let $\hat{Q}$ be an operator corresponding to a physical observable such that $\hat{Q}|0\rangle = 0|0\rangle$ and $\hat{Q}|1\rangle = 1|1\rangle$ Choose all of the following statements that are correct:

   (I) A bit cannot be in both 0 and 1 states. However, a qubit can be in a superposition of $|0\rangle$ and $|1\rangle$ states.

   (II) A measurement of observable $Q$ in a state $|q\rangle$ can only produce one of two possible values.

   (III) When you perform a measurement of Q in a qubit, you obtain one bit of information.

   (a) (I) and (II) only
   (b) (II) and (III) only
   (c) (I) and (III) only
   (d) All of the above

8. Choose all of the following that can form a qubit (i.e., that can be used as basis states for a qubit):

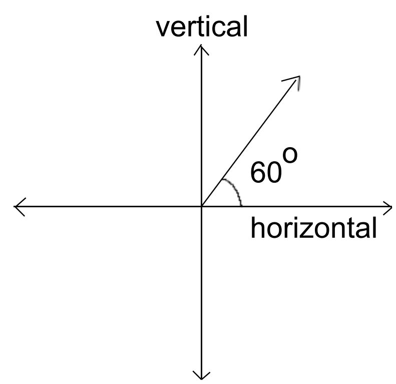
   (I) the eigenstates of $\hat{S}_z$ for a spin one-half particle

   (II) the two orthogonal polarization states of a photon

   (III) the ground state and the first excited state of a one dimensional infinite square well.

   (a) (I) and (II) only
   (b) (I) and (III) only
   (c) (II) and (III) only
   (d) (I), (II) and (III)

All of the angles (e.g., $60^0$ shown below) are measured with respect to the horizontal polarization axis toward the vertical polarization axis in the counterclockwise fashion. Assume that the polarizers in all questions do not reflect any light (they either absorb the light or let it pass through).




9. The vector space corresponding to polarization states of a photon is two dimensional. Suppose we choose the vertical and horizontal polarization states of a photon as the basis vectors. How can we represent a normalized $45^0$ polarization state?
   Notation: $|S\rangle_{45^0}$ represents $45^0$ polarization state, $|V\rangle$ represents the vertical and $|H\rangle$ represents the horizontal polarization state of a photon.
   (a) $|S\rangle_{45^0} = |V\rangle + |H\rangle$
   (b) $|S\rangle_{45^0} = |V\rangle - |H\rangle$
   (c) $|S\rangle_{45^0} = (|V\rangle + |H\rangle)/2$
   (d) $|S\rangle_{45^0} = (|V\rangle + |H\rangle)/\sqrt{2}$

10. Choose all of the following that can be used as basis vectors for the polarization state of a photon:

(I) vertical and horizontal polarization states

(II) $45^0$ and $-45^0$ polarization states

(III) $30^0$ and $-60^0$ polarization states

(a) (I) and (II) only
(b) (I) and (III) only
(c) (II) and (III) only
(d) (I), (II) and (III)

11. Choose all of the following pairs of basis vectors that are orthogonal to each other:

(I) vertical and horizontal polarization states

(II) $45^0$ and $-45^0$ polarization states

(III) $30^0$ and $-60^0$ polarization states

(a) (I) and (II) only
(b) (I) and (III) only
(c) (II) and (III) only
(d) (I), (II) and (III)

12. Choose all of the following statements that are correct:

(I) $0^0$ polarization is the same as $180^0$ polarization.

(II) $120^0$ polarization is the same as $-60^0$ polarization.

(III) $45^0$ polarization is the same as $-45^0$ polarization.

(a) (I) and (II) only
(b) (I) and (III) only
(c) (I), (II) and (III)
(d) None of the above

13. In a two dimensional vector space, how many different orthogonal bases can you choose?

(a) 2
(b) 4
(c) $2 \times 2 + 1 = 5$
(d) an infinite number

14. If you are using polarizers with $45^0$ and $-45^0$ polarization axes to measure the polarization states of a photon, what may be the best choice for basis vectors?

(a) $|S\rangle_{45^0}$, $|S\rangle_{-45^0}$

(b) $|V\rangle$, $|H\rangle$

(c) $|S\rangle_{30^0}$, $|S\rangle_{-60^0}$

(d) None of the above

15. Which one of the following is the correct expression for a general polarization state of a photon in terms of the vertical and horizontal polarization states as the basis vectors?

(a) $|S\rangle = \alpha|H\rangle + \beta|V\rangle$ where $\alpha^2 + \beta^2 = 1$.

(b) $|S\rangle = \alpha|H\rangle + \beta|V\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$.

(c) $|S\rangle = \alpha|H\rangle \times \beta|V\rangle$ where $\alpha + \beta = 1$.

(d) $|S\rangle = \alpha|H\rangle \times \beta|V\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$.

16. Choose all of the following that can form a qubit (i.e., that can be used as basis vectors for a qubit):

(I) $\{|H\rangle\}$

(II) $\{|H\rangle, |V\rangle\}$

(III) $\{|S\rangle_{45^0} = (|V\rangle + |H\rangle)/\sqrt{2}, |S\rangle_{-45^0} = (|V\rangle - |H\rangle)/\sqrt{2}\}$

(a) (I) only

(b) (II) only

(c) (II) and (III) only

(d) (I), (II) and (III)

17. If a single photon with a vertical polarization state passes through a vertical polarizer, which one of the following is true?

(a) The photon will pass through the polarizer with a 100% certainty.

(b) 75% of the photons will pass through the polarizer.

(c) 50% of the photons will pass through the polarizer.

(d) None of the photon will pass through.

18. If a single photon with a horizontal polarization state passes through a vertical polarizer, which one of the following is true?

(a) The photon will pass through the polarizer with a 100% certainty.

(b) 75% of the photons will pass through the polarizer.

(c) 50% of the photons will pass through the polarizer.

(d) None of the photon will pass through.

19. If a single photon with a polarization state $|S\rangle_{45^0}$ passes through a vertical polarizer, which one of the following is true?
(a) 25% of the photons will pass through the polarizer.
(b) 45% of the photons will pass through the polarizer.
(c) 75% of the photons will pass through the polarizer.
(d) 50% of the photons will pass through the polarizer.

20. If a single photon with a polarization state $|S\rangle_{60^0}$ passes through a vertical polarizer, which one of the following is true?
(a) 30% of the photons will pass through the polarizer.
(b) 60% of the photons will pass through the polarizer.
(c) 25% of the photons will pass through the polarizer.
(d) 75% of the photons will pass through the polarizer.

21. If a single photon with a normalized polarization state $|S\rangle = \alpha|H\rangle + \beta|V\rangle$ passes through a vertical polarizer, which one of the following is true?
(a) The photon passes through the polarizer with a probability $|\alpha|$.
(b) The photon passes through the polarizer with a probability $|\beta|$.
(c) The photon passes through the polarizer with a probability $|\alpha|^2$.
(d) The photon passes through the polarizer with a probability $|\beta|^2$.

22. If a single photon with a normalized polarization state $|S\rangle = \alpha|H\rangle + \beta|V\rangle$ passes through a horizontal polarizer, which one of the following is true?
(a) The photon passes through the polarizer with a probability $|\alpha|$.
(b) The photon passes through the polarizer with a probability $|\beta|$.
(c) The photon passes through the polarizer with a probability $|\alpha|^2$.
(d) The photon passes through the polarizer with a probability $|\beta|^2$.

23. If a single photon with a polarization state $|S\rangle_{60^0}$ passes through a $45^0$ polarizer, which one of the following is true? (Hint: Choose your basis vectors to be $\pm 45^0$ polarization states.)
(a) 45% of the photons will pass through the polarizer.
(b) 7% of the photons will pass through the polarizer.
(c) 93% of the photons will pass through the polarizer.
(d) 15% of the photons will pass through the polarizer.

**Flying Qubits: Insecure Protocol for Key Distribution over a Public Channel using Polarization States of Single Photons**

In all of the discussion below, keep in mind that it is impossible for Alice and Bob to meet in person.

Alice and Bob want to generate a shared binary "key" (for encoding and decoding secret information) over a public channel (where a third party can eavesdrop on what is being sent).

Alice and Bob discuss the following protocol over the phone (a public channel over which a third party can eavesdrop):

• Alice will send to Bob single photons with either horizontal or vertical polarization randomly with equal probability.

• Bob will intercept the photons sent by Alice using a polarizer which is randomly oriented either horizontally or vertically with equal probability. Bob's ideal polarizer has a 100% efficient photodetector behind it. When a photon passes through his polarizer, the photodetector clicks.

• They agree to denote a horizontally polarized photon state as bit '1' and the vertically polarized photon state as bit '0'.

• Every time Alice sends a photon, she will alert Bob over the public channel that she sent a photon without telling him the polarization of the photon.

• They also decide that every time Bob detects a photon in his polarization measurement, he will send an email (which is a public channel) to Alice saying "I got it" without writing to her whether he was using a horizontal or vertical polarizer.

• They decide that every time Bob says "I got it" both Alice and Bob will record that bit as part of the key they are generating for encoding and decoding information.

• A photon can only be intercepted by one person's polarizer/detector system. If Eve intercepts a photon sent by Alice with her polarizer/detector system, she sends to Bob a replacement photon.

• If Eve is eavesdropping and intercepting the photons sent by Alice with her polarizer, she will have to send to Bob a replacement photon, otherwise Bob will not receive a photon when Alice alerted him and he will know someone has been tampering with the system.

• Assume that when Eve is eavesdropping and intercepting the photons sent by Alice with her polarizer, she is using a polarizer with a horizontal or a vertical polarization axis with equal probability (identical to Bob's strategy).

• Assume that the time it takes Eve to replace a photon is negligible so that Bob does not notice any time-lag if Eve intercepts Alice's photon and sends to Bob a replacement photon in its place.

24. Suppose Bob uses his polarizer with a vertical polarization axis and detects a photon. With what certainty can he infer the polarization state of the photon that Alice sent and what is it?
(a) 100% certainty, vertical polarization
(b) 100% certainty, horizontal polarization
(c) 50% certainty, vertical polarization
(d) 50% certainty, horizontal polarization

25. Suppose Bob uses a polarizer with a vertical polarization axis and detects no photon. With what certainty can he infer the polarization state of the photon that Alice sent?
(a) 100% certainty, vertical polarization
(b) 100% certainty, horizontal polarization
(c) 50% certainty, vertical polarization
(d) 50% certainty, horizontal polarization

26. Can Bob keep his polarizer fixed rather than switching between a vertical and a horizontal polarizer to get the same information about what Alice sent? Explain your reasoning.
(a) Yes.
(b) No.

27. Complete the third column of the following table based upon your responses above. The first column gives the polarization state of the polarizer Bob uses, the second column gives what Bob observes in the detector placed behind his polarizer and the third column gives what Bob can infer about the polarization state of the photon from his observations.

| Bob uses | Bob observes | Bob infers |
|---|---|---|
| $\|V\rangle$ | nothing | |
| $\|V\rangle$ | photon | |
| $\|H\rangle$ | nothing | |
| $\|H\rangle$ | photon | |

28. If Eve is eavesdropping and she is intercepting the photons that Alice sends by keeping her polarizer with a fixed vertical axis, can she tell the polarization state of each photon that Alice sends with 100% certainty? Explain. (Note: Eve had overheard what Alice and Bob had discussed on the phone)
(a) Yes
(b) No

29. Eve can tell the polarization state of each photon that Alice sends with 100% certainty. Can Eve generate a replacement photon with the correct polarization (same polarization as Alice's) and send it to Bob without him realizing that the photon did not come from Alice but from Eve? Explain

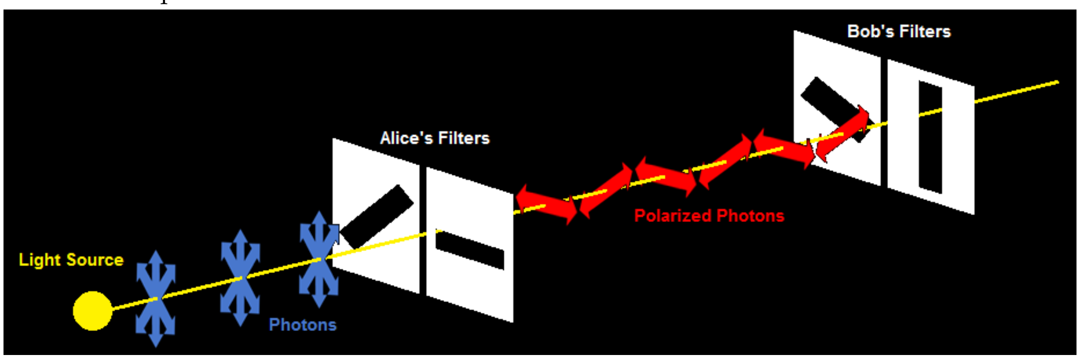
(a) Yes

(b) No

Summary: The above scheme is not safe for generating a secure key over a public channel for encoding and decoding information because Eve can generate the same key by intercepting the photons sent by Alice and sending to Bob replacement photons without Alice and Bob knowing.

**Flying Qubits: Secure Protocol for Key Distribution over a Public Channel using Two Non-orthogonal Polarization States of Single Photons**

Let's consider another protocol which is safe for generating a shared "key" for encoding and decoding information over a public channel using photons with non-orthogonal polarizations. This protocol is based on the BB84 protocol discussed in C. Bennett and G. Brassard, Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India, 175-179 (1984). However, the protocol discussed is a somewhat simplified version of BB84 in that Bob does not make use of a polarizing beam splitter and uses two non-orthogonal polarizers instead to intercept the photons sent by Alice. Thus, it takes twice as long to generate a key in the protocol discussed here compared to BB84.




Note: **All of the questions that follow in this tutorial refer to the following protocol.**

Alice and Bob discuss the following protocol over the phone (a public channel where a third party can eavesdrop):

• Alice will send single photons either with $+45^0$ polarization or $0^0$ polarization (horizontal polarization) randomly with equal probability.

• Bob will use a polarizer with polarization axis either at $-45^0$ or $90^0$ randomly with equal probability. The 100% efficient photo detector placed behind his polarizer detects every photon that passes through his polarizer.

• Over the phone they decide to label the $+45^0$ polarization state as bit "1" and the $0^0$ polarization state as bit "0".

• Every time Alice sends a photon, she alerts Bob on the phone that she is sending something but does not reveal its polarization.

• Every time Bob measures a photon in his photodetector (i.e., he gets a "click" in his photodetector), he sends Alice an email with "I got it" without saying "what" polarization he measured and both of them note down that "bit" as part of their shared key.

Note: **If a third person, Eve, tries to intercept the key distribution, we will assume that she uses the following protocol:**

• If Eve is eavesdropping and intercepting the photons sent by Alice, she does it with her polarizer with polarization axis either at $-45^0$ or $90^0$ with equal probability, identical to Bob's protocol.

• A photon can only be intercepted by one person's polarizer/detector system because it gets absorbed. If Eve intercepts a photon sent by Alice with her polarizer/detector system, she sends to

Bob a replacement photon. A photon in an arbitrary unknown state cannot be cloned according to No-Cloning Theorem.

• Eve sends to Bob a replacement photon even if the detector behind her polarizer does not click (and she does not know the polarization of the photon sent by Alice although she knows that Alice had alerted over a public channel that she was sending a photon); otherwise Bob will not receive a photon when Alice alerted him. If Eve did not send replacement photons to Bob for cases in which her detector did not click, the probability of Bob's detector (placed behind his polarizer) not clicking will be higher than if Eve was not eavesdropping. Therefore, Bob will know that someone was tampering with the system.

• Assume that the time it takes Eve to replace a photon is negligible so that Bob does not notice any time-lag if Eve intercepts Alice's photon and sends to Bob a replacement photon in its place.

30. Bob uses a $-45^0$ polarizer and detects a photon. In this case, with what certainty can Bob infer the polarization state of the photon that Alice sent and what is it?

(a) 100% certainty, $45^0$ polarization.

(b) 50% certainty, $45^0$ polarization.

(c) 100% certainty, $0^0$ polarization.

(d) 50% certainty, $0^0$ polarization.

31. Bob uses a $-45^0$ polarizer and the detector does not click. Can he infer the polarization state of the photon that Alice sent and what is it?

(a) Yes. Alice must have sent a photon with $0^0$ polarization.

(b) Yes. Alice must have sent a photon with $45^0$ polarization.

(c) No. Alice could have sent a photon with either polarization ($0^0$ or $45^0$).

(d) None of the above.

32. Bob uses a $90^0$ polarizer and detects a photon. With what certainty can Bob infer the polarization state of the photon that Alice sent and what is it?

(a) 100% certainty, $45^0$ polarization.

(b) 50% certainty, $45^0$ polarization.

(c) 100% certainty, $0^0$ polarization.

(d) 50% certainty, $0^0$ polarization.

33. Bob uses a $90^0$ polarizer and the detector does not click. Can he infer the polarization state of the photon that Alice sent and what is it?

(a) Yes. Alice must have sent a photon with $0^0$ polarization.

(b) Yes. Alice must have sent a photon with $45^0$ polarization.

(c) No. Alice could have sent a photon with either polarization ($0^0$ or $45^0$).

(d) None of the above.

34. Choose all of the following statements that are correct:

(I) If Bob's detector clicks, he can infer the polarization of the photon that Alice sent.

(II) If Bob's detector does not click, he cannot infer the polarization of the photon that Alice sent.

(III) If Alice sends a photon with $0^0$ polarization and Bob uses a $-45^0$ polarizer, that photon will be partly absorbed and partly transmitted.

**(a)** (I) only

**(b)** (II) only

**(c)** (I) and (II) only

**(d)** (I) and (III) only

35. Assume Alice and Bob conduct a large number of experiments. Complete the following table by recording in the third column "the probability that a photon will pass through and hence Bob's detector clicks": (Note: Please review the protocol for transmitting the key at the beginning of this section that Alice and Bob use.)

| | | Probability of detector click |
|---|---|---|
| Alice transmits $45^0$ | Bob uses $-45^0$ polarizer | |
| | Bob uses $90^0$ polarizer | |
| Alice transmits $0^0$ | Bob uses $-45^0$ polarizer | |
| | Bob uses $90^0$ polarizer | |

36. For which of the above situations in the table is Bob 100% sure about the polarization state of the photon sent by Alice?

(a) In all cases

(b) In none of the cases

(c) Only when a photon is blocked by his polarizer

(d) Only when a photon passes through his polarizer.

37. Choose all of the following statements that are correct:

(I) In 25% of all cases Alice will send a photon with $45^0$ polarization and Bob will use a $-45^0$ polarizer.

(II) In 25% of all cases Alice will send a photon with $0^0$ polarization and Bob will use a $-45^0$ polarizer.

(III) In 25% of all cases Alice will send a photon with $0^0$ polarization and Bob will use a $90^0$ polarizer.

**(a)** (I) and (II) only

**(b)** (I) and (III) only

**(c)** (II) and (III) only

(d) (I), (II) and (III)

38. For what percentage of his measurements is Bob 100% sure about the polarization of the photon sent by Alice out of all of the experiments that Alice and Bob conduct? Recall that Alice uses her polarizer with $45^0$ and $0^0$ polarization with equal probability and Bob uses his polarizer with $-45^0$ and $90^0$ polarization with equal probability.

(Hint: Use the table you constructed.)

(a) 75%

(b) 50%

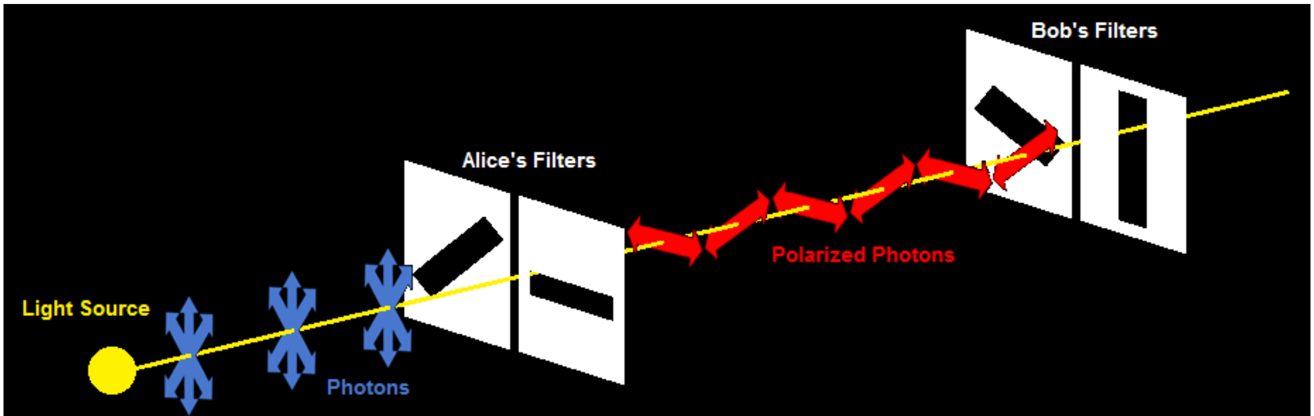
(c) 25%

(d) 12.5%

39. Complete the second last row of the following table, which refers to the situation when a bit is recorded by Alice and Bob as part of the key by writing Y (yes) and N (no). Complete the last row of the following table by writing the bit "1" or "0", which refers to what Alice and Bob record (if nothing is recorded and the "bit" is discarded, put a dash −).

(Note: Please review the protocol for transmitting the key at the beginning of this section that Alice and Bob use.)

| Alice's polarization: | ⤢ | ↔ | ⤢ | ↔ | ⤢ | ↔ |
|---|---|---|---|---|---|---|
| Bob's polarization: | ⤡ | ⤡ | ↕ | ↕ | ↕ | ⤡ |
| Bob's detector clicks: | N | N | Y | N | N | Y |
| Bit is recorded: (Y or N) | | | | | | |
| Which bit they record: (0 or 1 or -) | | | | | | |




| | | | | | | |
|---|---|---|---|---|---|---|
| **Alice's bit value** | 1 | 0 | 1 | 0 | 1 | 0 |
| **Alice's polarization** | / | — | / | — | / | — |
| **Bob's polarization** | \ | \ | \| | \| | \| | \ |
| **Bob's results** | N | N | Y | N | N | Y |
| **Transmitted key** | - | - | 1 | - | - | 0 |

| | | |
|---|---|---|
| Alice transmits 1 +45° | Bob measures with -45° filter | Photons always blocked |
| | Bob measures with 90° filter | Some photons blocked |
| | | Some photons pass |
| Alice transmits 0 +0° | Bob measures with -45° filter | Some photons pass |
| | | Some photons blocked |
| | Bob measures with 90° filter | Photons always blocked |

Check your answers to the previous questions about this quantum key sharing protocol between Alice and Bob with the information in the Figure on the previous page.

40. If Eve is eavesdropping and is using her polarizer with $-45^0$ or $90^0$ polarization axis randomly with equal probability to intercept the photon that Alice sent (same strategy as Bob), for what percentage of her measurement is she 100% sure about what Alice sent?
(a) 75% of the time.
(b) 50% of the time.
(c) 25% of the time.
(d) 12.5% of the time.

41. When Eve's detector does not click after she intercepts Alice's photon, she is not sure about the polarization of the photon sent by Alice. Eve will have to guess the polarization of the replacement photon she will send to Bob (in place of the one she intercepted). Suppose Eve is intercepting every photon sent by Alice and sending a replacement photon to Bob. Choose all of the following statements that are correct about the strategy for sending replacement photons to Bob when Eve is not sure what she intercepted from Alice:

(I) If Eve's polarizer is at $90^0$ when she intercepted the photon sent by Alice and the detector does not click, she has less chance of making an error if she sends a replacement photon with $0^0$ polarization to Bob.
(II) If Eve's polarizer is at $-45^0$ when she intercepted the photon sent by Alice and the detector does not click, she has less chance of making an error if she sends a replacement photon with $45^0$ polarization to Bob.
(III) If Eve's polarizer is at $90^0$ and the detector does not click, she has the same chance of making an error regardless of whether she sends $0^0$ or $45^0$ polarization photon to Bob to replace it.

**(a)** (I) only
**(b)** (II) only
**(c)** (III) only
**(d)** (I) and (II) only

42. Suppose Eve intercepts all the photons sent by Alice. Using the suggestions in the previous question, Eve uses the strategy described above to send the replacement photons to Bob. Out of all the replacement photons she sends to Bob, for what percentage of measurements will she make an error in sending the correct bit (same as that sent by Alice) to Bob?
(a) 75 % of the time.
(b) 50 % of the time.
(c) 25 % of the time.
(d) 12.5% of the time.

43. Suppose Alice transmits a photon at $45^0$ (i.e., "1") and Eve intercepts it with a polarizer at $90^0$ and the photon gets blocked. Eve does not know whether Alice had sent the photon with $45^0$ or $0^0$ polarization. In order to send a replacement photon to Bob, she makes a wrong guess and sends him a photon with $0^0$ polarization. Suppose Bob had his polarizer at $-45^0$ and there is a "click". Bob sends an email to Alice with the message "I got it" and both write down that "bit" as part of the key. Which one of the following is true?
(a) Alice will write a "1" while Bob will write a "0".
(b) Alice will write a "0" while Bob will write a "1".
(c) Both Alice and Bob will write a "1".
(d) Both Alice and Bob will write a "0".

44. After the whole key is generated, Alice and Bob decide to compare over the phone every $10^{th}$ bit to ensure they agree on the shared key they generated. (Of course, they discard the bits they compare so that it is not a part of the shared key they generate.) If they find a significant discrepancy in the bits they compare, it may be because

(I) Eve was eavesdropping and replacing some of Alice's photons with new photons having the other polarization.
(II) Alice noted the bits incorrectly.
(III) Bob noted the bits incorrectly.

Choose all of the above statements that are very likely:
(a) (I) only
(b) (II) only
(c) (III) only
(d) (I), (II) and (III)

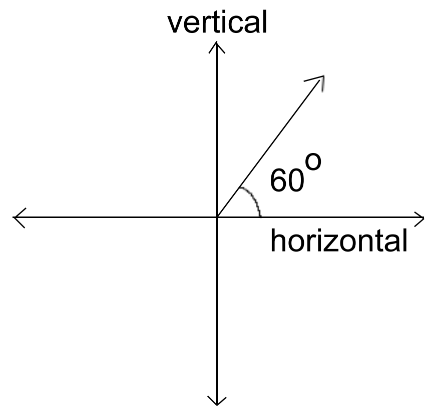
Considering Alice and Bob were not tired (typically these are machines), if they find a sufficiently large discrepancy, they will suspect eavesdropping. They will discard the key they generated over the public channel completely and decide to try to generate it at a later time (when nobody is eavesdropping).

**Summary:** There is no classical method for transmitting a key (for encoding and decoding information) securely over a public channel. We discussed a quantum mechanical protocol for transmitting a key securely over a public channel using the polarization states of single photons. Alice uses a polarizer with two non-orthogonal polarization states to send the photons (e.g., $0^0$ and $45^0$) and Bob uses a polarizer with two non-orthogonal polarization states (e.g., $90^0$ and $-45^0$) together with a photo detector to detect the photons sent by Alice. As noted earlier, this protocol is based on the BB84 protocol discussed in C. Bennett and G. Brassard, Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India, 175-179 (1984). However, the protocol discussed is a somewhat simplified version of BB84 in that Bob does not make use of a polarizing beam splitter and uses two non-orthogonal polarizers instead to intercept the photons sent by Alice. Thus, it takes twice as long to generate a key in the protocol discussed here compared to BB84.

In the original BB84 protocol, a systematic comparison after a sufficiently long key is generated by each person will display at least 25% error if someone was eavesdropping no matter how innovative the protocol of the eavesdropper is. Note that error can be introduced in the shared key generated by Alice and Bob due to decoherence (interaction of the photon with the surroundings which can change its polarization state or lead to scattering or absorption of photon). The effect of decoherence can be neglected (decoherence is relatively small over small distances). For example, quantum key distribution schemes, similar in spirit to the above protocol, have been tested successfully and the highest bit rate system currently demonstrated exchanges secure keys at 1 Mbit/s over 20 km of optical fiber and 10 kbit/s over 100 km of fiber. Applications are envisioned for the banking industry (e.g., http://www.idquantique.com/). For larger distances, decoherence effects will become important and quantum error correction will be needed. Quantum error correction techniques are challenging and have been tested only at a very rudimentry level. The main challenge is the necessity to correct the error introduced by decoherence without performing a measurement because a measurement can change the quantum mechanical state of the photon.

**Post-test**

All of the angles (e.g., $60^0$ shown below) are measured with respect to the horizontal polarization axis toward the vertical polarization axis in the counterclockwise fashion. Assume that polarizers do not reflect any light (they either absorb light or let it pass through).




Alice sends single photons randomly either with $+60^0$ polarization or $0^0$ polarization (horizontal polarization) with equal probability to Bob. Bob uses a polarizer with polarization axis oriented either at $-30^0$ or $90^0$ (vertical) randomly with equal probability. Behind his polarizer is a 100% efficient photodetector which detects every photon that passes through the polarizer (assume that the only source of photons is the one that Alice is using). Alice and Bob each know the protocol for sending single photons (i.e., Bob knows that Alice is sending single photons randomly either with $+60^0$ polarization or $0^0$ polarization with equal probability to him). Alice and Bob conduct a very large number of such experiments.

Use the above protocol to answer the following questions:

1. Bob uses the polarizer with a $-30^0$ polarization and his photo detector does not click. Choose all of the following statements that can be inferred based upon the above protocol used by them:

   (I) Bob is 100% sure about the polarization of the photon sent by Alice.

   (II) Alice must have sent a photon with a $+60^0$ polarization.

   (III) Alice must have sent a photon with a $+0^0$ polarization.

   **(a)** (I) only
   **(b)** (II) only
   **(c)** (III) only
   **(d)** None of the above

2. Bob uses a polarizer with a $-30^0$ polarization and his photo detector clicks. Choose all of the following statements that can be inferred based upon the above protocol used by Alice and Bob:

   (I) Bob is 100% sure about the polarization of the photon sent by Alice.

   (II) Alice must have sent photon with a $+60^0$ polarization.

   (III) Alice must have sent photon with a $+0^0$ polarization.

   **(a)** (II) only
   **(b)** (III) only
   **(c)** (I) and (II) only
   **(d)** (I) and (III) only

3. Suppose Alice transmits a photon with her polarizer set to $60^0$. Bob uses a $90^0$ polarizer to intercept it. Write down the probability that the photon will pass through Bob's polarizer.

4. Alice transmits a photon with $0^0$ polarization and Bob uses a $-30^0$ polarizer. Which one of the following statements is true?
   (a) The photon is blocked by his polarizer with a 100% certainty.
   (b) 75% of the photons will pass through the polarizer.
   (c) 25% of the photons will pass through the polarizer.
   (d) The photon will pass through with a 100% certainty.

5. Bob uses a $90^0$ polarizer and the detector does not click. Can he infer the polarization state of the photon that Alice sent? If so, what is it?
   (a) Yes. Alice must have sent a photon with $0^0$ polarization.
   (b) Yes. Alice must have sent a photon with $60^0$ polarization.
   (c) No. Alice could have sent a photon with either polarization ($0^0$ or $60^0$).
   (d) None of the above.

6. Choose all of the following statements that are correct based upon the protocol described:
   (I) Whenever Bob's detector clicks, he can infer the polarization of the photon that Alice sent.
   (II) Whenever Bob's detector does not click, he cannot infer the polarization of the photon that Alice sent.
   (III) If Alice sends a photon with $0^0$ polarization and Bob uses a $-30^0$ polarizer, that photon will be partly absorbed and partly transmitted.
   **(a)** (I) only
   **(b)** (II) only
   **(c)** (I) and (II) only
   **(d)** (I) and (III) only

7. Complete the third column of the following table by recording "the probability that a photon will pass through and hence Bob's detector clicks":

| | | Probability of detector clicking |
|---|---|---|
| Alice transmits $60^0$ | Bob uses $-30^0$ polarizer | |
| | Bob uses $90^0$ polarizer | |
| Alice transmits $0^0$ | Bob uses $-30^0$ polarizer | |
| | Bob uses $90^0$ polarizer | |

8. Using the table above for the case described in the preceding question, calculate the percentage of measurements in which Bob is 100% sure about the polarization of the photon that Alice sent out of all of the experiments that Alice and Bob conduct.