Warm up for Quantum Measurement

1. Energy eigenfunctions (stationary state wave functions) are eigenfunctions of

A. The momentum operator.
B. The Hamiltonian operator.
C. The position operator.
D. All of the above because all these eigenfunctions are proportional to each other.

2. Suppose $\psi_n(x)$ is an energy eigenfunction of a Hamiltonian $\hat{H}$. Choose all of the following statements that are correct.

(1) $\psi_n(x)$ must satisfy an eigenvalue equation $\hat{H}\psi_n(x) = E_n\psi_n(x)$

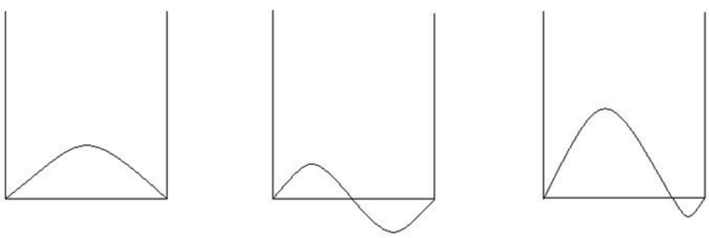
(2) $\psi_n(x)$ must satisfy the Time Independent Schrödinger Equation (TISE).

(3) When the system is in the state $\psi_n(x)$, the energy of the system must be well-defined. (Well-defined value of an observable in a given state implies that measurement of that observable will yield a particular value with 100% probability.)

A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. 2 and 3 only
E. All of the above

3. Choose the wave functions from the figures below that are energy eigenfunctions for an electron in a 1D infinite square well. (Assume the curves in graphs (I) and (II) are sinusoidal. The wavefunction in graph (III) is the normalized linear superposition of the wavefunctions in (I) and (II), i.e., $\Psi(x,t) = \frac{1}{\sqrt{2}}[\Psi_1(x,t) + \Psi_2(x,t)]$)

A. only (I)
B. only (I) and (II)
C. only (I) and (III)
D. all of them



(I) (II) (III)

4. Suppose $|\psi_1\rangle$ and $|\psi_2\rangle$ are eigenstates of a Hamiltonian $\hat{H}$ with energy eigenvalues E_1 and E_2. Which of the following is correct?

A. $\hat{H}\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right) = \frac{1}{\sqrt{2}}(E_1 + E_2)\cdot\left(|\psi_1\rangle + |\psi_2\rangle\right)$

B. $\hat{H}\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right) = \frac{1}{2}(E_1 + E_2)\cdot\left(|\psi_1\rangle + |\psi_2\rangle\right)$

C. $\hat{H}\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right) = \frac{1}{\sqrt{2}}(E_1|\psi_1\rangle + E_2|\psi_2\rangle)$

D. $\hat{H}\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right) = \frac{1}{2}(E_1|\psi_1\rangle + E_2|\psi_2\rangle)$

5. In the previous problem (question 4), is the state $\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)$ an eigenstate of $\hat{H}$? Explain.

6. Sketch the ground state and the first excited state energy eigenfunctions of a particle interacting with a simple harmonic oscillator (SHO) potential well. Is the shape of the ground state energy eigenfunction of the SHO the same as that for the ground state of the 1-D infinite square well? Explain.

7. Choose all of the following statements that are correct.

(1) Any wavefunction for a system can be expressed as a linear superposition of the energy eigenstates.

(2) Any normalized superposition of the energy eigenstates is a new energy eigenstate.

(3) Any normalized superposition of the energy eigenstates would satisfy the TISE.

A. only 1

B. only 3

C. 1 and 2

D. 1 and 3

E. All of the above

8. Explain in words at least three characteristics of the energy eigenstates and their relation to the Hamiltonian of the system.

9. Which one of the following is the mathematical representation of a **position** eigenstate for an electron in an **infinite** square well in the position representation? Suppose x_0 is inside the well.

A. $\Psi(x) = A\delta(x - x_0)$, where A is a suitable constant.

B. $\Psi(x) = A(x - x_0)$

C. $\Psi(x) = A\sin(n\pi x / a)$, where a is the width of the well.

D. $\Psi(x) = Ae^{ikx}$

10. Draw a graphical representation of the position eigenfunction with eigenvalue x_0 for an electron in an **infinite** square well.

11. Would your answer to the previous problem (*question 10*) be different if you were asked to draw a graphical representation of the position eigenfunction with eigenvalue x_0 for an electron interacting with a **finite** square well or simple harmonic oscillator (**SHO)** potential energy well? Explain.

12. Suppose $\psi(x)$ is an eigenfunction of the position operator $\hat{x}$ with eigenvalue x_0. Choose all of the following statements that are correct.

    (1) $\hat{x}\psi = x_0\psi$

    (2) $\hat{x}\psi = (x - x_0)\psi$

    (3) $\hat{H}\psi = E\psi$

A. only 1
B. only 2
C. 1 and 3
D. 2 and 3
E. All of the above

13. Rewrite the eigenvalue equation in *question 12* with the actual form of the position eigenfunction (in the position representation) with eigenvalue x_0.

14. Choose all of the following statements that are correct about an electron in an eigenstate of the position operator.
    (1) The position of the electron is well defined in this state.
    (2) The momentum of the electron cannot be well defined in this state due to the uncertainty principle.
    (3) The measurement of position will yield a definite value with 100% certainty.

A. only 1
B. 1 and 2
C. 1 and 3
D. 2 and 3
E. All of the above

15. Consider the following statement: "The position eigenstate and energy eigenstate are the same for a given system. After all, they are all eigenstates." Explain why you agree or disagree with this statement.

16. Explain in words at least three characteristics of position eigenstates.

Suppose $\hat{A}$ is the operator corresponding to a physical observable A. $\psi_a(x)$ is an eigenfunction of $\hat{A}$ with eigenvalue a. Write down the eigenvalue equation for the operator $\hat{A}$ with eigenvalue a.

17. Choose all of the following statements that are correct about an electron in an eigenstate of the operator $\hat{A}$. ($\hat{A}$ is the operator corresponding to a physical observable A)

(1) The value of the physical observable A is well defined in this state.

(2) If $\hat{A}$ commutes with the Hamiltonian $\hat{H}$ and the eigenvalue spectra of the operators $\hat{A}$ and $\hat{H}$ are non-degenerate, then the energy of the system is well defined in this state.

(3) The measurement of observable A in this state will yield a definite value with 100% certainty.

A. only 1

B. only 3

C. 1 and 2

D. 2 and 3

E. All of the above