Reflective Homework

The first two reflective homework problems refer to the free particle system: The generic form for a wave packet for a free particle at time $t = 0$ is given by

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx} dk \tag{1}$$

(1) At time $t = 0$, you construct two different Gaussian wave packets for the free particle:
One is $\Psi_1(x, t = 0) = A_1 e^{-x^2/(2\sigma_1^2)}$ which can be written as $\Psi_1(x, t = 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_1(k) e^{ikx} dk$ and another is $\Psi_2(x, t = 0) = A_2 e^{-x^2/(2\sigma_2^2)}$ which can be written as $\Psi_2(x, t = 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi_2(k) e^{ikx} dk$. Here A_1 and A_2 are the normalization constants and σ_1 and σ_2 are the standard deviations of the two wave packets respectively. Assume $\sigma_1 > \sigma_2$. Using the position-momentum uncertainty relation, explain qualitatively how the width of the functions $\phi_1(k)$ and $\phi_2(k)$ will differ when you form the two Gaussian wave packets.
(2) The generic form for a wave packet for a free particle at time t is given by $\Psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \omega(k)t)} dk$. where $\omega(k)$ is the angular frequency which is a function of wave vector k. Suppose $\phi(k)$ in the above equation is not highly localized about any particular k value. What can you say about the group velocity of the wave packet in that scenario?
(3) Explain the difference between a particle in a classical bound state and a quantum mechanical bound state. What is the main criterion for distinguishing between these two types of bound states?
(4) Draw an example of a particle interacting with a potential energy which is classically in a bound state but quantum mechanically in a scattering state. Is it possible to have a particle interacting with a potential energy which is quantum mechanically in a bound state but classically in a scattering state? Explain.