Test B for Larmor Precession

All of the following questions refer to the following system:

An electron is in an external magnetic field B which is pointing in the z direction. The Hamiltonian for the electron spin is given by $\hat{H} = -\gamma B \hat{S}_z$ where γ is the gyromagnetic ratio and $\hat{S}_z$ is the z component of the spin angular momentum operator.

Notation: $\hat{S}_z \left|\uparrow\right\rangle_z = \frac{\hbar}{2}\left|\uparrow\right\rangle_z$, *and* $\hat{S}_z \left|\downarrow\right\rangle_z = -\frac{\hbar}{2}\left|\downarrow\right\rangle_z$.

For reference, the eigenstates of $\hat{S}_x$ *and* $\hat{S}_y$ *are given by:*

$$\left|\uparrow\right\rangle_x = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_z + \left|\downarrow\right\rangle_z\right), \quad \left|\downarrow\right\rangle_x = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_z - \left|\downarrow\right\rangle_z\right)$$

$$\left|\uparrow\right\rangle_y = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_z + i\left|\downarrow\right\rangle_z\right), \quad \left|\downarrow\right\rangle_y = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_z - i\left|\downarrow\right\rangle_z\right)$$

1. If the electron is initially in an eigenstate of $\hat{S}_y$, does the expectation value of $\hat{S}_y$ depend on time? Justify your answer.

2. If the electron is initially in an eigenstate of $\hat{S}_y$, does the expectation value of $\hat{S}_x$ depend on time? Justify your answer.

3. If the electron is initially in an eigenstate of $\hat{S}_y$, does the expectation value of $\hat{S}_z$ depend on time? Justify your answer.

4. Consider the following statements from Andy and Caroline when the electron is initially in an eigenstate of $\hat{S}_y$ (the y component of the spin angular momentum):

Andy: The electron will NOT be in an eigenstate of $\hat{S}_y$ forever because the state will evolve in time.

Caroline: I disagree. If a system is in an eigenstate of an operator corresponding to a physical observable, it stays in that state forever unless a perturbation is applied.

With whom do you agree? Explain.

A. Andy
B. Caroline

5. If the electron is initially in an eigenstate of $\hat{S}_z$, does the expectation value of $\hat{S}_y$ depend on time? Justify your answer.

6. If the electron is initially in an eigenstate of $\hat{S}_y$, is there any precession of $\langle \vec{S} \rangle$ about the z axis? If your answer is yes, explain why and give an example of a situation where there will be no precession of $\langle \vec{S} \rangle$ about the z axis. If your answer is that there is no precession in the given case, explain why.