**Understanding the Mach-Zehnder Interferometer (MZI) with Single Photons: Homework 2**

**The goals of this homework are to use a simplified ideal version of MZI to help you:**

* **Connect qualitative understanding of the MZI with a simple mathematical model**

1. **Determine the product space of path states and polarization states for a polarized photon in the U and L paths**
2. **Determine the matrix representation in a given basis of the quantum mechanical operators that correspond to beam splitter 1, beam splitter 2, mirrors, phase shifter, and polarizers**
3. **Find the effect of various quantum mechanical “time-evolution” operators on an input state and the probability of a detector D1 or D2 clicking for four cases:**
   1. **Original MZI setup with a phase shifter in the upper path**
   2. **Horizontal polarizer in upper path, phase shifter in upper path, and vertical polarizer in lower path**
   3. **Horizontal polarizer in upper path, phase shifter in upper path, vertical polarizer in lower path, and 45 degree polarizer placed in the lower path between BS2 and detector D1**
   4. **Horizontal polarizer and phase shifter in upper path**

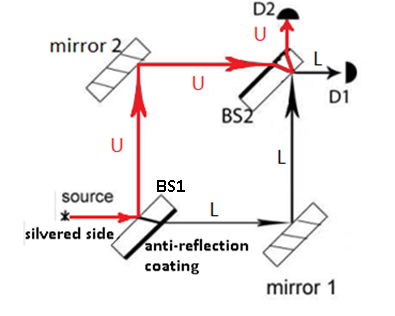
The setup for the ideal Mach-Zehnder interferometer (MZI) shown below is as follows:

* All angles of incidence are 45° with respect to the normal to the surface.
* For simplicity, we will assume that a photon can only reflect from one of the two surfaces of the identical half-silvered mirrors (beam splitters) BS1 and BS2 because of anti-reflection coatings.
* The detectors D1 and D2 are point detectors located symmetrically with respect to the other components of the MZI as shown.
* The photons originate from a monochromatic coherent point source. (Note: Experimentally, a source can only emit nearly monochromatic photons such that there is a very small range of wavelengths coming from the source. Here, we assume that the photons have negligible “spread” in energy.)
* Assume that the photons propagating through both the U and L paths travel the same distance in vacuum to reach each detector.
* In the classical usage of the Mach-Zehnder Interferometer, a “beam” of light is sent which gets separated spatially after passing through BS1. Now, we will send one photon at a time through the MZI.
* In all of the discussions below, ignore the effect of polarization of the photons due to reflection by the beam splitters or mirrors.
* Assume that beam splitters BS1 and BS2are infinitesimally thin so that there is no phase shift when a photon propagates through them.
* For the entire tutorial, assume that a large number () of photons are sent one at a time.



Before we begin, we will make a few assumptions:

* In all of the matrix representations of the operators, in a given basis, we will simplify the “” sign or “is represented in a given basis by” with “” for convenience.
* The beam splitters are 50/50 splitters, meaning that a measurement of the photon position immediately after it exits the beam splitter BS1 would yield an outcome such that the photon is either in the upper path or the lower path with 50% probability.
* The silvered side of the beam splitter is the point of reflection. No reflection occurs at the air-glass interface (the bold side of the beam splitter), due to anti-reflection coatings.
* From here on, assume that the thickness of the beam splitters is negligible so the phase shift introduced by the propagation of light through the beam splitters is zero ().
* No relative phase shift is introduced when a photon propagates through vacuum because the photon travels the same distance in vacuum along each of the U and L paths.
* The upper path is marked in RED. The lower path is marked in BLACK.
* When determining the matrix operators in the two dimensional Hilbert space for the photon path states, assume the basis vectors are taken in the order , .
* When determining the matrix operators in the two dimensional Hilbert space for the photon polarization states, assume the basis vectors are taken in the order , .
* When determining the matrix operators in the four dimensional product space involving both the photon path states and polarization states, assume the basis vectors are taken in the order , , , .

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**Figure 1**

Table 1: Phase shifts of a photon state due to reflection, transmission, and propagation through medium

|  |  |  |
| --- | --- | --- |
|  | Initially in medium with lower *n* | Initially in medium with higher *n* |
| Reflection at interface | Phase shift of π | No phase shift |
| Transmission at interface | No phase shift | No phase shift |
| Propagation through a medium | Phase shift φ depends on thickness and refractive index *n* | |

1. **Determine the product space for a polarized photon in the U and L paths**

* You have already learned that the photon states corresponding to the and paths can be represented as two linearly independent states, such as and , since the Hilbert space is two dimensional. The and states are orthogonal, e.g., and normalized, e.g.,
* Any operator in a two dimensional Hilbert space can be represented by a matrix in the chosen basis.
* When polarizers are added, we must consider the Hilbert space corresponding to the polarization state of the photon.
* The Hilbert space involving both path states (from the U and L paths) and polarization states is a product space.
* Let’s choose a basis in which we denote the polarization state of the vertically polarized photon to be and the polarization state of the horizontally polarized photon to be . These two polarizations are linearly independent and all other photon polarizations can be constructed from these states, e.g., . The Hilbert space for the polarization of the photon is also two dimensional (similar to the Hilbert space for the path states). The and polarization states are orthogonal, e.g., and normalized, e.g., and .
* The product space of the polarization states and and the path states and is four dimensional. There are four possible basis states in the product space:, , , and .
* To determine the matrix representation for the photon states, , , and , we find the basis states in product space (tensor product of path states and polarization states which takes us from two dimensional Hilbert spaces to a four dimensional Hilbert space):

* Let’s define a tensor product of two general two-dimensional vectors and as

1. Keeping this in mind, find the matrix representation for the four possible photon states , , , and using , ,, and .
2. Consider the following conversation between two students about a +45° polarized photon emitted from the source shown in Figure 1.

* Student 1: If the photon is emitted from the source in the upper path (as shown in Figure 1) with +45° polarization, then the initial photon state can be denoted like this:

.

* Student 2: I disagree with you. The final state, , you found corresponds to a source emitting a photon with -45° polarization. Actually, the state of a +45° polarized photon in the upper path should look like this:

.

With whom do you agree? Explain your reasoning.

**B. Determine the matrix representation of the quantum mechanical operators that correspond to beam splitter 1, beam splitter 2, mirrors, phase shifter, and polarizers**

1. **Determining the matrix representation of beam splitters 1 and 2**

* Previously, you found that the matrix representations for the quantum mechanical operators corresponding to beam splitter 1 and beam splitter 2 in the two dimensional Hilbert space (only taking into account the photon path states) are

[BS1]=

[BS2]=

assuming the basis vectors are taken in the order , .

* Now we need to determine what these operators look like in the four dimensional product space (taking into account both the photon path states and polarization states).
* Beam splitter 1 and beam splitter 2 only affect the path states and , NOT the polarization states and of the photon.
* We will always use the following convention for the order of the basis vectors when determining all the matrices corresponding to the optical elements (BS1, BS2, mirrors, phase shifter) and polarizers in the product space: , , , .
* Predict the various matrix elements corresponding to beam splitter 1 below in the boxes (Hints are provided below to find these matrix elements).

* In homework 1, you learned that BS1 reflects the upper path state of the photon by π, so there is a phase shift of the upper path state by . The lower path state is not phase shifted. Thus,

.

* The beam splitters do not affect the polarization state of the photon, i.e., and .
* Therefore, the matrix elements and are

and

.

* The matrix elements and (because of the orthogonality of and , e.g., and in the polarization state subspace). (Note that the operator [BS1] does not affect the polarization state of the photon.) So the matrix elements that mix different polarizations and are and

.

* Thus, we can fill in the upper quadrant like this:

1. Keeping in mind the phase shifts for the photon path states and in the setup given in Figure 1 and the orthogonality conditions of the path states and and polarization states and , fill in the rest of the matrix for operator [BS1] shown above in the product space of path states and polarization states.

* The matrix representation of beam splitter 1 that you should have determined in the previous question is

If this does not match your answer to the previous question, go back and check your work.

1. Consider the following conversation between two students about the [BS1] matrix using the basis vectors in the order , , , .

* Student 1: When I calculated the matrix elements of [BS1], I obtained  
  . This makes sense because there is along the diagonal in the, subspace (or upper left quadrant), which means that the component of the photon state in the upper path has been phase shifted by π.
* Student 2: I agree with you. And in the other quadrants, there are 1’s along the diagonal. This means that [BS1] does not lead to any phase change in those subspaces. In particular, [BS1] does not change the phase of the photon state in the , subspace. Actually, we can observe a relationship between the [BS1] matrix in the space involving only the photon path states and the [BS1] matrix in the product space involving both photon path states and polarization states as follows:

.

Do you agree with the students? Explain your reasoning.

* Now we need to find the matrix representation for the operator corresponding to beam splitter 2. Predict the matrix corresponding to the operator for beam splitter 2 in Figure 1 which does not affect polarization. (Hints will be provided below to answer this question.)

1. Keeping in mind the phase shifts from Table 1, determine the action of [BS2] on the upper and lower path states:
2. Using the orthogonality conditions and for the corresponding two-dimensional subspace, fill in the matrix for [BS2].

* The matrix representation of [BS2] that you should have determined in the previous question is

If this does not match your answer to the previous question, go back and check your work.

1. **Determining the matrix representations corresponding to the mirrors**

* Earlier, when you only considered photon path states, you found the matrix corresponding to both of the mirrors in Figure 1 as , where is the identity operator.

1. The mirror operator in a particular path only changes the path state of the photon by . It does not affect the polarization state. Keeping this in mind, write down the matrix corresponding to both of the mirror operators for the produce space involving both the photon path state and photon polarization state. Assume the basis vectors are taken in the order , , , .
2. **Determining the matrix representations corresponding to a phase shifter, e.g., a piece of glass placed in the upper or lower path**

* Earlier, when you only considered photon path states, you found the matrix corresponding to a phase shifter operator in the upper path as .

1. The phase shifter in a particular path only changes the path state of the photon by , in which is the phase shift due to the phase shifter. The phase shifter does not affect the polarization state of the photon. Keeping this in mind, write down the matrix corresponding to the phase shifter operator for a phase shifter placed anywhere in the upper path between BS1 and BS2. Assume the basis vectors are taken in the order , , , .
2. Write down the matrix corresponding to the phase shifter operator for a phase shifter placed in the lower path between BS1 and BS2.

* In summary, the matrices corresponding to the mirror operator and phase shifter operators in the upper or lower path in the product space that includes path and polarization state are
  + , where is the identity operator.
  + , where is the phase shift due to the phase shifter.
  + , where is the phase shift due to the phase shifter.

If these matrices do not match your answers to the previous questions, go back and check your work.

1. Consider the following conversation between two students about the representation of the phase shifter operators, and , assuming the basis vectors are taken in the order , , , .

* Student A: Placing a phase shifter in the upper path only affects the phase of the component of the photon state in the upper path. So it makes sense that the operator , since in the , subspace we have the matrix element along the diagonal. We see 1’s along the diagonal in the , subspace because the phase shifter does not act on the component of the photon state in the lower path, so in that subspace we have an identity operator.
* Student B: I agree with you. And a phase shifter operator in the lower path looks like because the phase shifter only acts on the component of the photon state in the lower path. We have the matrix elements along the diagonal in the , subspace. And we have 1’s along the diagonal in the , subspace (an identity operator) because the phase shifter does not act on the component of the photon state in the upper path. We can observe a relationship between the phase shifter matrix in the space involving only the photon path states and the phase shifter matrix in the product space involving both the photon path states and polarization states as follows:

.

Do you agree with the students? Explain your reasoning.

1. **Determining the matrix representations of the operators corresponding to polarizers**

* Let’s first recapitulate how a polarizer acts on the photon polarization states and in a two dimensional space before we consider the product space which also includes the photon path states and .
* We will define the matrix representing a vertical polarizer as and the matrix representing a horizontal polarizer as and we will use the following convention for the matrix representation of polarizer operators in the two dimensional Hilbert space (i.e., choose the states in the order , to write the matrix elements of and ):
* A vertical polarizer will allow a vertically polarized photon to pass through and will completely block a horizontally polarized photon. So we know that and

.

1. Consider the following statement from Student 1:

* Student 1: The matrix corresponding to the vertical polarizer operator is , since the matrix elements are

Do you agree with Student 1? Explain your reasoning.

1. In the preceding question, Student 1’s reasoning is correct for finding the matrix elements for the vertical polarizer operator in the given basis. Using the information above, write down the matrix corresponding to the horizontal polarizer operator for the two dimensional polarization Hilbert space assuming that the basis vectors are chosen in the order , .

* Now you will determine the matrix corresponding to a +45° polarizer operator, . The normalized state of a +45° polarized photon can be written as an equal superposition of the states and as follows:

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* The state of a -45° polarized photon can be written as

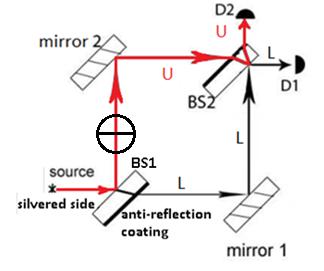
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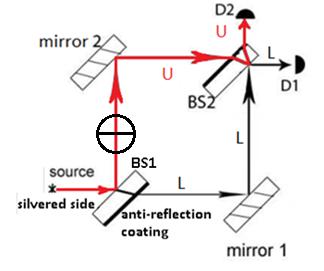
* A +45° polarizer, , will allow a +45° polarized photon to pass through and will completely block a -45° polarized photon. So and .

1. Which one of the following matrices represents the +45° polarizer operator, , if the basis vectors are chosen in the order , ?
3. Which one of the following matrices represents the -45° polarizer operator, , if the basis vectors are chosen in the order , ?

* In summary, if the basis vectors are chosen in the order , , then
  + The matrix corresponding to the vertical polarizer is .
  + The matrix corresponding to the horizontal polarizer is .
  + The matrix corresponding to the +45° polarizer is .
  + The matrix corresponding to the -45° polarizer is .

If your answers to the preceding questions do not match these, go back and check your work.

* Now you will find the matrices corresponding to the vertical, horizontal, and +45° polarizer operators in the four dimensional product space by including both photon polarization and path states.
* Suppose we change the original MZI setup by placing a horizontal polarizer in the upper path, as follows:
* The horizontal polarizer in the upper path will only affect the component of the photon state in the upper path. It will block the vertical polarization component of the photon state in the upper path and will let the horizontal polarization component of the photon state in the upper path pass through. It will not affect the component of the photon state in the lower path.
* We will always use the following convention for the order in which the basis vectors are chosen to determine the matrices for the polarizer operators in the product space: , , , .

1. Consider the following conversation between students about the matrix corresponding to a horizontal polarizer operator, in the upper path for the setup shown in the figure above (basis vectors are chosen in the order , , , ):

* Student 1: The matrix corresponding to a horizontal polarizer operator in the upper path is

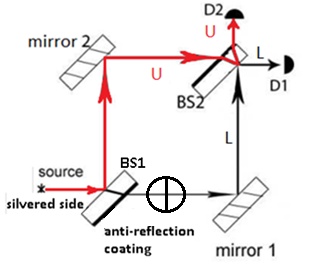
, since the only matrix element that survives is . We can see that and because the operator acting on the L path state must be zero.

* Student 2: I disagree with you. The matrix corresponding to a horizontal polarizer in the upper path should look like because and . The operator acting on the L path state of the photon must not change the L path state, so returns the same state back, as follows: and . is an identity operator in the L path subspace (lower right quadrant) of the full matrix for the setup shown above.
* Student 3: I think the matrix corresponding to a horizontal polarizer in the upper path should look like , since the horizontal polarizer only allows the horizontally polarized photons to pass through.

With whom do you agree? Explain your reasoning.

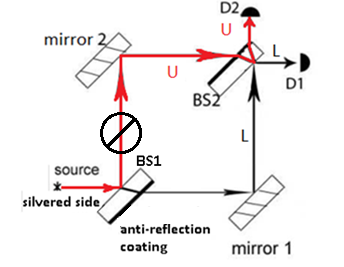
If you did not agree with Student 2 in the preceding question, go back and check your work.

1. Which one of the following matrices represents the horizontal polarizer operator, in the lower path if the basis vectors are chosen in the order , , , ?

* Suppose we place a vertical polarizer in the lower path of the MZI, like this:
* The vertical polarizer in the lower path will only affect the component of the photon state in the lower path. It will block the horizontal polarization component of the photon state in the lower path and will let the vertical polarization component of the photon state in the lower path pass through.

1. Which one of the following matrices represents the vertical polarizer operator, in the lower path (shown above) if the basis vectors are chosen in the order , , , ?
2. Write down the matrix corresponding to a vertical polarizer operator in the upper path, , assuming that the answer to the previous question is (c).

* Suppose we place a +45° polarizer in the upper path, like this:

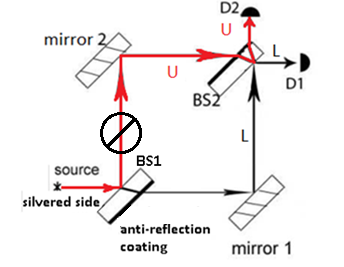


* In the two dimensional space, you already determined that the matrix corresponding to the +45° polarizer operator is in the , basis.
* The +45° polarizer in the upper path will affect the component of the polarization state of a photon in the upper path and will not affect the component of the polarization state of a photon in the lower path.

1. Choose the matrix which represents the +45° polarizer operator in the upper path, , assuming that the basis vectors are chosen in the order , , ,

Hint: When you act with the +45° polarizer operator, , on a photon in the upper path with +45° polarization, i.e., , it should allow the photon to pass through without any change, as follows::

. On the other hand, should block a photon in the upper path with -45° polarization, i.e., .

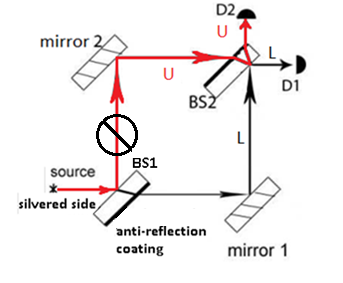


1. Consider the following conversation between two students about the answer to the preceding questions:

* Student A: I believe that the matrix corresponding to the +45° polarizer operator in the upper path, as shown in the figure above, should look like . The +45° polarizer operator only changes the photon state in the subspace of the U path state.
* Student B: I agree with you. And in the subspace of the L path state (lower right quadrant), the +45° polarizer operator in the upper path, , acts as an identity operator because it does not change the component of the photon state in the L path.

Do you agree with the students? Explain your reasoning.

1. Now write down the matrix corresponding to a +45° polarizer operator in the lower path, , if basis vectors are chosen in the order , , ,

* Suppose we place a -45° polarizer in the upper path, as follows:

1. Choose the matrix which represents the -45° polarizer operator in the upper path, , assuming that the basis vectors are chosen in the order , , , .

Hint: When you act with the -45° polarizer operator in the upper path on the photon state in the upper path with -45° polarization, i.e., , it should allow the photon to pass through without any change, as follows:

. On the other hand, it should block a photon in the upper path with +45° polarization, i.e., .

1. Write down the matrix corresponding to a -45° polarizer operator in the lower path, , if basis vectors are chosen in the order , , , .
2. Consider the following conversation between students:

* Student 1: I believe the polarizer operators we have found are both hermitian and unitary.
* Student 2: I disagree. The polarizer operators are not unitary because they do not preserve the norm of the state. It makes sense that they do not preserve the norm because the polarizer absorbs a polarization component of the photon state. However, since we are not interested in the component absorbed, it is OK to consider these polarizer operators in our analysis even though they are not unitary, time-evolution operators.
* Student 3: We can also check unitarity by , where is the hermitian conjugate of the operator . We will find that polarizer operators will, in general, not satisfy this relation.
* Student 4: There is no particular reason for the polarizer operators to be hermitian either because they do not correspond to a physical observable, e.g., position, momentum, energy, etc. If a polarizer operator turns out to be hermitian in a given case, it is just a fluke.

With whom do you agree? Explain your reasoning.

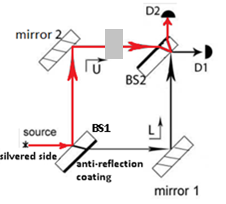
1. So far, you have determined the basis vectors , , , and and matrix representations for the operators corresponding to beam splitter 1 , beam splitter 2 , mirrors , phase shifter , and polarizers . Using your answers to the previous questions, fill out the following tables for the four dimensional product space. Assume the basis vectors are taken in the order , , , .

|  |  |
| --- | --- |
| Basis vector | Matrix representation |
|  |  |
|  |  |
|  |  |
|  |  |

|  |  |
| --- | --- |
| Operator | Matrix Representation in the product space including both the path and polarization states |
|  |  |
|  |  |
|  |  |

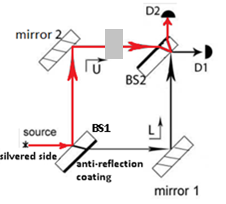
|  |  |  |
| --- | --- | --- |
|  | Matrix representation of operator involving the upper path | Matrix representation of operator involving the lower path |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. **Find the effect of the quantum mechanical operators on an input state of a photon and the probability of a detector D1 or D2 clicking (and registering a photon) when the source emits +45° polarized photons**
   1. **Original MZI setup with a phase shifter in the upper path**

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* We will now find the action of BS1, mirrors, phase shifter in upper path, and BS2 on the state of a single 45° polarized photon emitted from the source in the upper path state , i.e.,

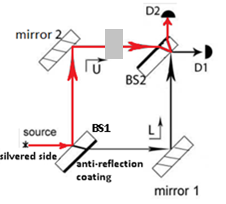
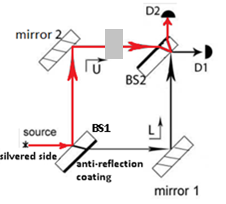
, as shown in the figure above.

1. Predict the photon state just before BS2 after it has propagated through BS1, i.e., and select one of the following answers that matches your prediction.
2. , where A is a normalization constant.
3. , where A is a normalization constant.
4. , where A is a normalization constant.
5. , where A is a normalization constant.
6. ****Consider the following conversation between two students when the source emits +45° polarized photons:

* Student 1: The photon state after it has propagated through BS1 should be a superposition of the upper and lower path states, i.e., , where A is a normalization constant. Since the upper path state is phase shifted by π, it should be multiplied by . The lower path state is not phase shifted. Also, BS1 does not affect the photon polarization, so the photon remains +45° polarized after passing through BS1.
* Student 2: I disagree. The photon state after it has propagated through BS1 should be , where A is a normalization constant.
* Student 3: You are both saying the same thing because a +45° polarized photon in the upper path is represented mathematically as . A +45° polarized photon in the lower path is represented mathematically as . Thus,

, where .

With whom do you agree?

1. . Describe in words what this output represents.
2. Predict the state of the photon just before BS2 after it has propagated through BS1, the mirrors, and the phase shifter in the upper path and select one of the following answers that match your prediction.
3. , where A is a normalization constant.
4. , where A is a normalization constant.
5. , where A is a normalization constant.
6. , where A is a normalization constant.
7. The correct answer to the previous question is (c), in which . After the photon exits BS2, the final state of the photon is

Consider the following statement about the photon state in the product space of both the path and polarization states.

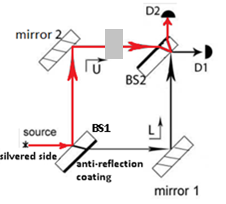
* Student 1: The photon state is equivalent to

because we have chosen our basis vectors to be , , , and .

Therefore,

.

Do you agree with student 1?

In the figure to the right, the final state of the photon before it reaches the detectors is

.

Detector D1 projects the component of the photon state along the L path. Thus, the total probability of detector D1 registering a photon is .

Detector D2 projects the component of the photon state along the U path. Thus, the total probability of detector D2 registering a photon is .

1. Consider the following conversation between two students:

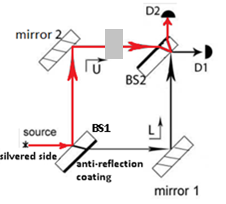
* Student A: The total probability of detector D1 clicking above was determined by squaring the coefficients of and state and adding them. You have chosen a basis for the polarization subspace to be , . The total probability of detector D1 clicking would depend on what polarization basis you choose.
* Student B: No. The total probability of detector D1 registering a photon does not depend on what polarization basis you choose. Even if you write the state of the photon above in terms of another orthogonal polarization basis, e.g., and , you would obtain the same total probability.

Which student you agree with? Write the and states in terms of and in state in the product space above. Then, square the coefficients and add them to obtain the total probability of detector D1 registering a photon. Does your prediction agree with your answer? If not, reconcile the difference between your prediction and observation.

1. Consider the following conversation between two students:

* Student A: The probability that detector D1 registers a photon is . But what about the polarization state of the photon when it is registered at detector D1? Is the photon detected a horizontally polarized photon, a vertically polarized photon, or neither?
* Student B: Actually, the polarization of the photon is not well defined, i.e., when detector D1 clicks, the photon is in a superposition of vertical and horizontal polarizations. We have made a partial measurement of path state because detector D1 projects the lower path. But we cannot tell what type of polarization the photon has when it was detected in detector D1. If you also want to measure polarization, you must place a polarizer in front of detector D1.

Do you agree with Student B’s explanation? Explain your reasoning.

1. Consider the following conversation between two students, assuming that Student B was correct in the previous question:

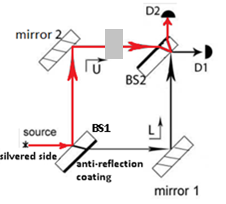
* Student A: Let’s cover detector D1 in the lower path with a horizontal polarizer. Now we can measure both the path state and polarization state of the photon. The probability that detector D1 in the lower path registers a horizontally polarized photon is
* Student B: I see. And if we want to determine the probability that detector D1 in the lower path registers a vertically polarized photon, we would place a vertical polarizer in front of detector D1. The probability that detector D1 registers a vertically polarized photon is
* Student A: And notice that when you add the two probabilities together, you get the total probability of detector D1 registering a photon, . This makes sense because we chose two orthogonal polarization bases, and , in which to make a measurement.

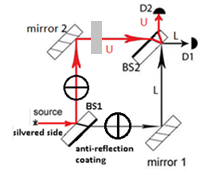
Do you agree with the students? Explain your reasoning.

1. Consider the following conversation between two students:

* Student A: If the total probability of detector D1 registering a photon in the figure shown is , what does this tell us about whether or not we have “which-path” information?
* Student B: In this case, we don’t have “which-path” information since the probability depends on the phase difference of the photon state from the two paths . This means that the single photon interferes with itself. BS2 evolves the superposition state of the photon in such a way that the photon state from both U and L paths can be projected by each detector. Thus, when a photon arrives at a detector, the phase difference between the U and L paths in the superposition state at a detector will lead to interference of the photon with itself.

With whom do you agree? Explain your reasoning.

1. Suppose the phase shifter is removed (). What is the total probability that detector D1 clicks? Is this probability consistent with what you learned in the earlier part of the Mach Zehnder Interferometer tutorial (Recall that without the phase shifter, detector D1 corresponds to constructive interference in the given setup)?
2. Suppose the phase shifter is removed and . What is the total probability that detector D2 clicks? Is this probability consistent with what you learned in the earlier part of the Mach Zehnder Interferometer tutorial (Recall that without the phase shifter, detector D2 corresponds to destructive interference in the given setup)?
3. **Horizontal polarizer and phase shifter in upper path and vertical polarizer in lower path**



* We will now find the action of BS1, horizontal polarizer in the upper path, vertical polarizer in the lower path, mirror, phase shifter in the upper path, and BS2 as shown in the figure above on a single 45° polarized photon emitted from the source in the upper path state, i.e., .

1. Consider the following conversation between two students:

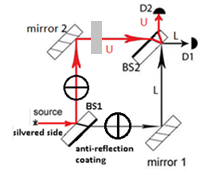
* Student 1: When we act on the photon state with a vertical polarizer in the lower path and a horizontal polarizer in the upper path as shown above, to find out how the state evolves we can use the matrices corresponding to the polarizer operators we found previously, i.e., and .
* Student 2: I agree with you. But we can also combine the two matrices and into one matrix by multiplying them, , since the vertical polarizer operator in the lower path and horizontal polarizer operator in the upper path commute with one another. Then, we can act on the single photon state after the photon passes through BS1 with the combined matrix , which is equivalent to acting on the photon state with and separately one after another in any order.

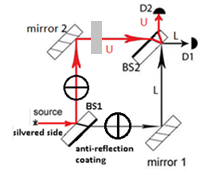
Do you agree with Student 2’s explanation? Explain your reasoning.

1. Consider the following conversation between two students:

* Student 1: In the setup shown above, would the phase difference between the U and L path states matter in detectoring what happens when a photon arrives at a detector at the end as shown above?
* Student 2: No. Since we have “tagged” the photons by placing orthogonal polarizers in the U and L paths, we have “which-path” information about the photon when it is registered at a detector. It is useless to calculate the phase difference between the photon state from the U and L paths for information about interference because we have “which-path” information about each photon that arrives at detectors D1 or D2.
* Student 3: I agree with you. And the probability for a detector registering a photon will not depend on the phase of the phase shifter. Each detector will register a photon with equal probability.

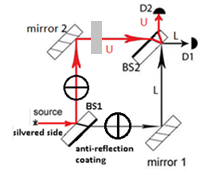
Do you agree with Student 2 and Student 3? Explain your reasoning.

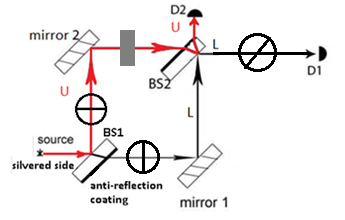
1. In the preceding question, Student 2 and Student 3 are correct in their predictions. We will now verify their statements. In the figure shown, predict the state of the photon just before BS2 after it has propagated through BS1, horizontal polarizer in the upper path and vertical polarizer in the lower path, mirrors, and phase shifter in upper path and select one of the following answers that match your prediction.
2. , where A is a normalization constant.
3. , where A is a normalization constant.
4. , where A is a normalization constant.
5. , where A is a normalization constant.
6. The correct answer to the previous question is (d), in which which you can find by acting on the initial state with the “time-evolution” operators in this order: . Describe in words what the mathematical output in the preceding question represents.



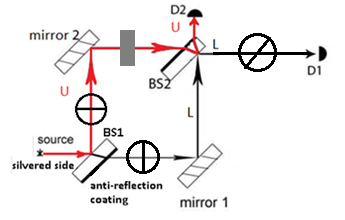
1. After the photon exits BS2, the state of the photon is

.

1. Which one of the following correctly describes the photon state as given above?
2. Based upon your answer to the previous part, what is the total probability of detector D1 clicking and registering a photon (if it is not covered by a polarizer)? Does the probability depend on the phase shifter?
3. If detector D1 in the lower path is covered by a vertical polarizer, what is the probability that detector D1 clicks and registers a vertically polarized photon? If detector D1 is covered by a horizontal polarizer, what is the probability that detector D1 clicks and registers a horizontally polarized photon?
4. What is the total probability that detector D2 clicks (if it is not covered by a polarizer)? Does the probability depend on the phase shift of the phase shifter?
5. If detector D2 in the upper path is covered by a vertical polarizer, what is the probability that detector D2 clicks and registers a vertically polarized photon? If detector D2 is covered by a horizontal polarizer, what is the probability that detector D2 clicks and registers a horizontally polarized photon?
6. Suppose the phase shifter is removed (). What is the final state of the photon right before the detectors? What is the total probability that detector D1 clicks? What is the total probability that detector D2 clicks? Are these probabilities consistent with what you learned in the earlier part of the Mach Zehnder Interferometer tutorial (Recall that without the phase shifter, the setup was such that there was completely constructive interference at detector D1 and completely destructive interference at detector D2)?
7. **QUANTUM ERASER with a horizontal polarizer in upper path, vertical polarizer in lower path, phase shifter in upper path, and 45 degree polarizer placed in the lower path between BS2 and detector D1**

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* We will now find the action of BS1, horizontal polarizer in the upper path, vertical polarizer in the lower path, mirror, phase shifter in upper path, BS2, and a 45° polarizer in the lower path (between BS2 and detector D1) on a single 45° polarized photon emitted from the source in the upper path state shown above, i.e., when is the initial state of the photon emitted from the source.
* The MZI setup shown above is a quantum eraser because the +45° polarizer before detector D1 erases “which-path” information about the photon arriving at detector D1. In the following questions, you will check this erasure of “which-path” information due to the 45° polarizer mathematically.

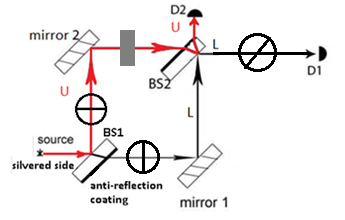
1. Predict the state of the photon just before BS2 after it has propagated through BS1, the polarizers, mirrors, and the phase shifter in the upper path and select one of the following answers that match your prediction.
2. , where A is a normalization constant.
3. , where A is a normalization constant.
4. , where A is a normalization constant.
5. , where A is a normalization constant.
6. The state of the photon after it has propagaed through BS1, the polarizers, mirrors, and the phase shifter in the upper path is . In terms of the product space of path states and polarization states, what is , where basis vectors are chosen in the order , , , and ?
7. The correct answer to the preceding question is (d). For the setup shown above, the final state of the photon is

.

1. Consider the following conversation between two students about the output shown above:

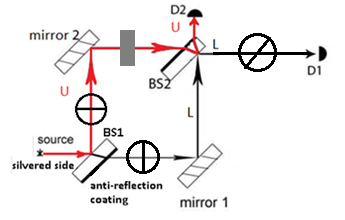
* Student 1: The total probability of detector D1 registering a photon is since detector D1 projects the component of the photon state along the L path.
* Student 2: I agree. If detector D1 in the lower path is covered by a vertical polarizer, the probability of detector D1 registering a vertically polarized photon is , in which is the final state of the photon given above. Thus,

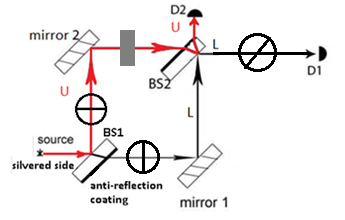
.

And if detector D1 is covered by a horizontal polarizer, the probability of detector D1 registering a horizontally polarized photon is , where is the final state of the photon shown above. Thus,

.

Do you agree with Student 1 and Student 2? Explain your reasoning.

1. Does the total probability that detector D1 “clicks” (and registers a photon) in the preceding question depend on the phase shift of the phase shifter, e.g., changing the thickness of the piece of glass? In particular, as we gradually change the thickness of the glass piece, will we observe the probability of registering a photon at detector D1 change? Explain your reasoning.
2. Is “which-path” information known about the photon when detector D1 “clicks” in the preceding question? Explain your reasoning.
3. What is the total probability that detector D2 registers a photon if detector D2 is not covered by a polarizer in the preceding question?
4. In the figure above, does the probability that detector D2 “clicks” (and registers a photon) depend on the phase shift of the phase shifter, e.g., changing the thickness of the piece of glass? Explain your reasoning.
5. Is “which-path” information known about the photon when detector D2 “clicks”? Explain your reasoning.
6. Suppose the phase shifter is removed (). What is the final state of the photon right before detector D1 and D2? What is the total probability that detector D1 clicks (and registers a photon) if detector D1 is not covered a polarizer? What is the total probability that detector D2 clicks (and registers a photon) if detector D2 is not covered by a polarizer? Is this probability consistent with what you learned in the earlier part of the Mach Zehnder Interferometer tutorial (recall that without the phase shifter, the setup was such that there was completely constructive interference at detector D1 and completely destructive interference at detector D2)?



1. Consider the following conversation between two students about the MZI setup shown above:

* Student 1: The final state of the photon right before the detectors in the figure shown above is

. We can rewrite this equation as , since is a superposition of vertical and horizontal polarization components that yields and . For figuring out what arrives at detector D1 in the lower path, we only need to look at the component of the state along the L path. This implies that the vertical and horizontal polarization components of the photon arrive in phase as in the lower path before entering the 45° polarizer between BS2 and detector D1. Therefore, the 45° polarizer allows all photons to pass through instead of absorbing them.

* Student 2: However, when a photon is registered at detector D1, the photon is in a superposition state from the U and L paths and we cannot tell which path the photon took. Thus, interference would be observed at detector D1.
* Student 3: I see. But what if we change the polarization of the 45° polarizer to 90°? Would it still be a quantum eraser setup?
* Student 1: No. If we change the polarization of the 45° polarizer to 90°, the final state of the photon is . Only the vertical component of the photon state from the lower path can be projected in detector D1. So that setup is not a quantum eraser.
* Student 2: However, if the polarization axis of the 45° polarizer was changed to 30°, it would absorb some photons but allow some photons to pass through (less than if the polarization axis was at 45°). Thus, we can project the photon components from the U and L path states into detector D1 and we do not have “which-path” information for the photon. Thus, this setup would be a quantum eraser. Interference will be observed at detector D1.

Do you agree with student 1 and student 2’s explanations? Explain your reasoning.

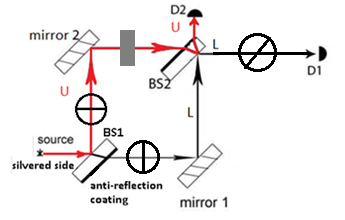
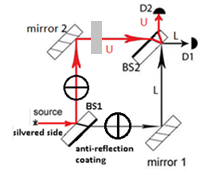
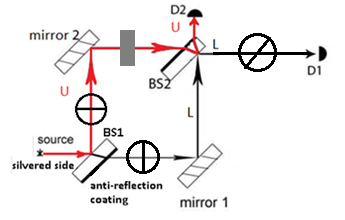


Figure 1 Figure 2

1. Consider the following conversation between two students about the MZI setups shown above.

* Student A: The final state of the photon in figure 1 is and the final state of the photon in figure 2 is . How can we tell that figure 1 is not a quantum eraser setup and figure 2 is a quantum eraser by looking at the the final state of the photon?
* Student B: In figure 1, the probabilities of a photon arriving at detectors D1 and D2 do not depend on the phase shift of the phase shifter. So of all of the photons that make it through the polarizers, half of them arrive at each detector D1 and D2. On the other hand, in figure 2, the total probability of the photon arriving at detector D1 depends on the phase shift of the phase shifter. So we could observe constructive interference, destructive interference, or intermediate interference at detector D1 depending on the phase shift of the phase shifter. However, in figure 2, since there is no 45° polarizer between BS2 and detector D2, we have “which-path” information for a photon that arrives at detector D2 and we would not observe interference at detector D2. From the expression given above for figure 2, the total probability of the photon arriving at detector D2 is and does not depend on the phase shift of the phase shifter.
* Student A: But when the phase shifter is removed (), the final states of the photon in each of the figures are the same, . So how can we tell that figure 2 is a quantum eraser case?
* Student B: For the special case when the phase shifter is removed, the states in both figures 1 and 2 look the same and we cannot tell that figure 2 is a quantum eraser if the source is emitting a highly collimated beam of single photons. If the source emits a highly collimated beam of a large number () of single photons, then the probability that detector D1 clicks is for both situations (figure 1 and figure 2) for . We cannot tell if the 45° polarizer is erasing quantum information just by considering one special case because we cannot tell if there is interference unless we change the phase difference between the two paths and observe that it makes a difference. However, if the source emits a large number () of single photons with a transverse Gaussian width and we replace detector D1 with a planar detector (screen) we would be able to tell that figure 2 is a quantum eraser setup. We would see an interference pattern emerge on the planar screen due to the path length differences of the photon with a transverse Gaussian width, as in the simulation.

Do you agree with Student B’s explanation? Explain your reasoning.



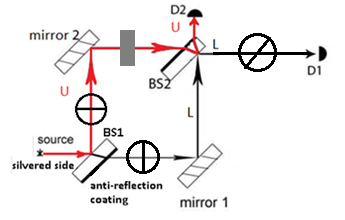
1. Consider the following conversation between students about the photon arriving at detector D2 in the MZI setup shown above:

* Student A: If in the figure above, we can write the final state of the photon

as . Does this mean that -45° polarized photons arrive at detector D2 via the upper path?

* Student B: No. If there is no polarizer placed in front of detector D2 in the upper path and a photon arrives at detector D2, we have made a partial measurement of only the photon path state but NOT the polarization state. The detection of a photon at detector D2 cannot tell us what is the polarization type of the photon registered. The detector simply “clicks” when a photon arrives. However, if we place a -45° polarizer between BS2 and detector D2, it would allow the photon with -45° polarization to pass through and the probability of the detector D2 clicking is .
* Student C: I agree with Student B. Also, if we place a +45° polarizer between BS2 and detector D2 in the upper path, it would block all photons because of the orthogonality of state, i.e., . Indeed, if you place a +45° or a -45° polarizer in front of the detector D2, a good polarization basis to write the final state of the photon is the one involving polarization states so that the state of the photon right before the detector is represented as .
* Student B: I agree with you. And if you place a vertical or horizontal polarizer in front of the detector D2, a good basis to write the final state of the photon arriving at detector D2 is in the basis, such as . Then you can see that if you place a vertical polarizer in front of detector D2 in the upper path, the probability of detector D2 registering a vertically polarized photon is . And, if you place a horizontal polarizer in front of detector D2 in the upper path, the probability of detector D2 registering a horizontally polarized photon in the upper path is .

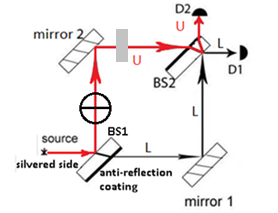
Do you agree with the students? Explain your reasoning.



1. In the figure above, suppose the source emits -45° polarized photons, . The final state of the photon is

.

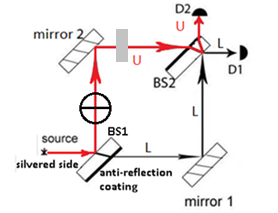
1. What is the total probability that detector D1 registers a photon (assume detector D1 is not covered by a polarizer) in the figure above.
2. Unpolarized light can be thought of as an equal mixture of 45° polarized photons and -45°polarized photons. To find the total probability of unpolarized photons arriving at detector D1, we can average the total probabilities of detector D1 registering a photon for the two cases in which the source emits +45° single photons and -45° single photons (questions 44 and 48 (a)). Does the probability of unpolarized photons arriving at detector D1 depend on the phase shift of the phase shifter? Does the quantum eraser setup make a distinction between polarized and unpolarized photons (will the quantum eraser work with unpolarizedphotons)?
3. **Horizontal polarizer and phase shifter in upper path**



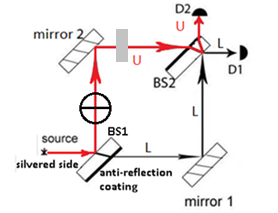
* In the figure above, we will now find the action of BS1, horizontal polarizer in the upper path, mirror, phase shifter in upper path, and BS2 on a single 45° polarized photon emitted from the source in the upper path state , i.e., when is the initial state of the photon emitted from the source.

1. For the figure above, predict the output of the photon state just before BS2 after the photon has propagated through BS1, the horizontal polarizer in the upper path, and reflected off the mirrors, i.e., what is ?
2. For the figure shown above, the photon state after the photon propagates through BS1, the horizontal polarizer in the upper path, and the mirrors is

. Describe in words what this output represents.

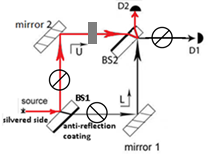
1. After the photon propagates through BS1, the horizontal polarizer in the upper path, mirrors, phase shifter in the upper path, and BS2, the photon state in the figure above is

.

1. What is the total probability that detector D1 clicks (if not covered by a polarizer)? If detector D1 is covered by a vertical polarizer, what is the probability that detector D1 registers a vertically polarized photon? If detector D1 is covered by a horizontal polarizer, what is the probability that detector D1 registers a horizontally polarized photon?
2. What is the total probability that detector D2 clicks (if not covered by a polarizer) in the figure shown? If detector D2 is covered by a vertical polarizer, what is the probability that detector D2 registers a vertically polarized photon? If detector D2 is covered by a horizontal polarizer, what is the probability that detector D2 registers a horizontally polarized photon?
3. For photons with which polarization type do you have “which-path” information? Based on the output

, do these photons have equal probability of arriving at each detector D1 and D2? Explain.

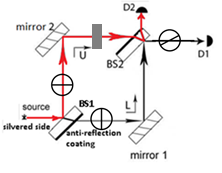
1. For photons with which polarization type do you NOT have “which-path” information? Based on the output , do these photons display interference at detectors D1 and D2? Explain.



1. Bonus: In the setup shown above, if the source emits 45° polarized single photons, the final state of the photon is

.

With what total probabilities do the detectors D1 and D2 register a photon (if they are not covered by polarizers)? Do you have “which-path” information about any photons emitted from the source that arrive at detectors D1 and D2?



1. Bonus: In the setup shown above, the polarizer between BS2 and detector D1 has a polarization axis of 30°. If the source emits 45° polarized single photons, the final state of the photon is

. With what total probabilities do the detectors D1 and D2 register a photon? Do you have “which-path” information about any photons emitted from the source that arrive at detectors D1 and D2?