Reflective Homework

(1) "The eigenstates of position are orthogonal to the eigenstates of energy". This statement is incorrect. The eigenstates of energy with different eigenvalues are orthogonal to each other but energy eigenstates are not orthogonal to position eigenstates. The energy eigenstates form a basis for the Hilbert space in which the state of the system lies.

(2) "If an electron in the one dimensional infinite square well is initially in a position eigenstate, the expectation value of an observable Q will be time-independent if and only if $[\hat{X}, \hat{Q}] = 0$". This statement is incorrect. There is nothing special about position eigenstates when it comes to issues related to time-dependence of expectation value. The only eigenstates that are special for issues related to time-dependence of wave function or the time-dependence of expectation values are the energy eigenstates (also called stationary states) because the Hamiltonian of the system governs the time-evolution of the system. If an observable Q is such that $[\hat{H}, \hat{Q}] = 0$, then the expectation value of Q will be time-independent. Also, note that if the initial state was not a position eigenstate but an energy eigenstate, the expectation value of all observables (with no explicit time-dependence) will be time-independent.

(3) "If the expectation value of an observable Q is zero in the initial state, then the expectation value $\langle \hat{Q} \rangle$ cannot have any time-dependence." This statement is incorrect. It is similar to the incorrect statement that "if the velocity of the particle is zero initially, then the rate of change of velocity (or acceleration) is zero". If that were true, no object could start from rest! The wave function could evolve in time and so the expectation value at future times may be non-zero even if the expectation value is zero in the initial state.

(4) A particle is interacting with a one dimensional harmonic oscillator potential energy well.
(a) If the particle is initially in a momentum eigenstate, the expectation value of an operator $\hat{Q}$ (no explicit time dependence) does not depend on time if $[\hat{H}, \hat{Q}] = 0$ (i.e., Q is a constant of motion). Note that the momentum eigenstates are not stationary states.

(b) If the particle is initially in an energy eigenstate, the expectation value of an operator $\hat{Q}$ (no explicit time dependence) does not depend on time because energy eigenstate is a stationary state.