

Reflective Homework

(1) “A magnetic field applied in the z direction cannot change the probability of measuring the different values of $\hat{S}_x$ (x component of spin angular momentum operator) because the field is orthogonal to the eigenstates of S_x ”. This statement is incorrect. The magnetic field is a vector in the physical space in which the experiment is performed. The eigenstates of $\hat{S}_x$ are vectors in an abstract Hilbert space. It does not make sense to talk about vectors in two different vector spaces being orthogonal to each other. Note that for the same reason it is not appropriate to say that the field is orthogonal to the eigenstates of the x component of the orbital angular momentum operator $\hat{L}_x$.

Notation: $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the orthonormal eigenstates of $\hat{S}_z$ (the z component of the spin angular momentum) of the electron.

(2) $\sqrt{3/7}|\uparrow\rangle + \sqrt{4/7}|\downarrow\rangle$ represents the state of the electron for the spin degree of freedom when you first measure S_z and then S_x in immediate succession. The possible values of S_x you will measure are $\hbar/2$ and $-\hbar/2$ with a probability $1/2$ each. This is because the first measurement of S_z collapse the system into an eigenstate of $\hat{S}_z$ (either $|\uparrow\rangle$ or $|\downarrow\rangle$) and each of the eigenstates of $\hat{S}_z$ is a superposition of eigenstates of $\hat{S}_x$ in which there is equal probability of measuring S_x .

(3) If after the measurement of S_x you measure S_z in immediate succession. For S_z , we will measure $\hbar/2$ and $-\hbar/2$ with a probability $1/2$ each. This is because the first measurement of S_x will collapse the system to an eigenstate of $\hat{S}_x$ and each of the eigenstates of $\hat{S}_x$ is a superposition of eigenstates of $\hat{S}_z$ in which there is equal probability of measuring S_z .

(4) Statement 1: “An electron spin cannot be in an eigenstate of the spin angular momentum operator $\vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$. In other words, you cannot measure the total spin angular momentum vector $\vec{S}$ of an electron”. This statement is correct because different components of spin angular momentum vector do not commute with each other which implies that we cannot find a complete set of simultaneous eigenstates of the different components of spin angular momentum operator. Thus, we cannot be in a state in which all components of the spin angular momentum vector are well-defined. (An analogous statement is true for the orbital angular momentum operator.)

Statement 2: “I agree but we can measure the magnitude of the spin angular momentum vector”. This statement is correct because the square of the magnitude of the spin angular momentum operator $\hat{S}^2$ commutes with each component of the spin angular momentum vector.

(5) If you make measurements of “the distance of the electron from the nucleus” on an ensemble of hydrogen atoms all in the ground state, we do not expect to find all electrons at the same distance from the nucleus. Based upon the wave function, there is a certain probability of finding the electrons at various distance from the nucleus.

(6) Suppose the state of the system for a one-dimensional infinite square well ($0 \leq x \leq a$) is an equal linear superposition of the ground state and first excited state as follows: $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ at time $t = 0$.

The probability of measuring position between x and $x + dx$ at time $t = 0$ in Dirac notation is $|\langle x|\psi\rangle|^2 dx = (|\langle x|\psi_1\rangle + \langle x|\psi_2\rangle|^2)/2 dx$.

The probability of measuring position between x and $x + dx$ at time $t = 0$ in the position representation is $|\psi(x, t = 0)|^2 dx = |(\psi_1(x) + \psi_2(x))|^2/2 dx$.

The probabilities of measuring energies E_1 and E_2 respectively at time $t = 0$ are $1/2$ each (in Dirac notation $|\langle\psi_1|\psi\rangle|^2$ and $|\langle\psi_2|\psi\rangle|^2$ respectively and in the position representation they are $\int_{x=0}^{x=a} dx \psi_1^*(x)\psi(x, t = 0)$ and $\int_{x=0}^{x=a} dx \psi_2^*(x)\psi(x, t = 0)$ respectively). The probability is zero of measuring any other energy in the given quantum state at $t = 0$.

The wave function at time t is $|\psi(t)\rangle = (|\psi_1\rangle e^{-iE_1 t/\hbar} + |\psi_2\rangle e^{-iE_2 t/\hbar})/\sqrt{2}$.

Thus, the probability of measuring position between x and $x + dx$ at time $t > 0$ in Dirac notation is $|\langle x|\psi(t)\rangle|^2 dx = |\langle x|\psi_1\rangle e^{-iE_1 t/\hbar} + \langle x|\psi_2\rangle e^{-iE_2 t/\hbar}|^2/2 dx$. Note that this probability is different from that at time $t = 0$ due to the time-dependent factors.

The probability of measuring position between x and $x + dx$ at time $t > 0$ in the position representation is $|\psi(x, t)|^2 dx = |(\psi_1(x) e^{-iE_1 t/\hbar} + \psi_2(x) e^{-iE_2 t/\hbar})|^2/2 dx$.

The probabilities of measuring energies E_1 and E_2 respectively at time $t > 0$ are still $1/2$ each (in Dirac notation $|\langle\psi_1|\psi(t)\rangle|^2$ and $|\langle\psi_2|\psi(t)\rangle|^2$ respectively and in the position representation they are $|\int_{x=0}^{x=a} dx \psi_1^*(x)\psi(x, t)|^2$ and $|\int_{x=0}^{x=a} dx \psi_2^*(x)\psi(x, t)|^2$ respectively where the time-dependent phase factors cancel out). The probability is zero of measuring any other energy in the given quantum state at $t > 0$.