Solution to the Reflective Homework

1) Since the potential energy is infinite for $x < 0$, the possible wave function for the system must be zero at $x = 0$. The solutions for the time-independent Schroedinger equation for a half harmonic oscillator are a subset of the solutions for the full harmonic oscillator (since the problem for $x > 0$ is identical for both full and half harmonic oscillators). This means that the energies for a half harmonic oscillator must be the odd n energies of the corresponding full harmonic oscillator (recall that the even quantum numbers denoting energies of the full harmonic oscillator n=0,2,4,6...etc. do not have stationary state wave functions that go to zero at $x = 0$).

2) For both the infinite square well and the simple harmonic oscillator, in the limit as n goes to infinity, the difference in the energy per unit energy is zero. For example, $\lim_{n\to\infty}(E_{n+1} - E_n)/E_n = \lim_{n\to\infty}(2n+1)/n^2 = \lim_{n\to\infty} 2/n = 0$ for the infinite square well and $\lim_{n\to\infty}(E_{n+1} - E_n)/E_n = \lim_{n\to\infty} 1/n = 0$ for a harmonic oscillator. This result is one manifestation of the correspondence principle. The discreteness of the energy levels is not important for large n states and the results of quantum mechanics will tend towards the classical results.

3) The method for finding the wave function at time t for a non-stationary state wave function for a harmonic oscillator and for an infinite square well are similar. You should first write the wave function for the system at time $t = 0$ in terms of a linear superposition of stationary states. Then, at time t, each of the terms in the superposition will develop a time-dependent phase factor $e^{-iE_n t/\hbar}$.

4) The expectation values of none of these observables (or any time-independent observable) depend on time in a stationary state. That is a defining property of a stationary state.