Solutions to Reflective Homework

(1) We consider four normalized vectors representing the state of a quantum system in a two-dimensional Hilbert space (e.g., spin of an electron): $\binom{1}{0}$, $\binom{-1}{0}$, $\binom{i}{0}$ and $\binom{-i}{0}$.
If we measure a physical observable Q, the probability of measuring different values of Q in each of these states is same **in general** because all these states only differ by an overall phase factor and the overall phase cannot affect the probabilities of measuring different values of Q. For example, $\binom{-1}{0} = \binom{1}{0} \times e^{i\pi}$ and $\binom{i}{0} = \binom{1}{0} \times e^{i\pi/2}$.

(2) We consider four unnormalized vectors representing the state of a quantum system in a two dimensional Hilbert space (e.g., spin of an electron): $\binom{1}{1}$, $\binom{-1}{1}$, $\binom{-i}{1}$ and $\binom{i}{1}$.
If we measure a physical observable Q, the probability of measuring different values of Q in each of these states is different **in general** because these states differ by more than an overall phase factor. For example, $\binom{-1}{1} \neq \binom{1}{1} \times e^{i\phi}$ where ϕ is an angle.

(3) A person comments that x^n is linearly independent of 1, x, x^2,...$x^{(n-1)}$ because $x^n \neq a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} x^{(n-1)}$ where $a_i, i = 0, 1, ..$ are any complex numbers. Another person claims that x^n is not linearly independent of $x^{(n-1)}$ because you can construct x^n from $x^{(n-1)}$ by using $x^n = x^{(n-1)} \times F(x)$ where $F(x) = x$ is a linear function. The first person is correct and the second person is incorrect from the definition of linearly independent vectors in a vector space.

(4) In an N dimensional vector space, a set of N linearly independent vectors that span the space is called a "basis". The basis is not unique for a given vector space. For example, for a one-dimensional infinite square well, the vector space in which the wavefunction lies (Hilbert space) is infinite dimensional. The energy eigenstates form one set of basis vectors for this space and position eigenstates form another set of basis vectors for the same infinite dimensional vector space. Another example is a problem involving a car on an inclined plane in classical mechanics in which two different choice of basis vectors (co-ordinate axes) could be choosing axes along vertical and horizontal directions or choosing the axes to be along the inclined plane and perpendicular to it.

(5) Two people describe the SAME state of a system $|\Psi\rangle$ in a two dimensional vector space by different column vectors. Person 1 writes column vector $\binom{a}{b}$ to describe $|\Psi\rangle$ and Person 2 writes column vector $\binom{a'}{b'}$ to describe $|\Psi\rangle$. In the expression each person writes to describe $|\Psi\rangle$, $a \neq a'$ and $b \neq b'$. Also, the first column vector <u>cannot</u> simply be written as the second column vector times a phase factor $e^{i\theta}$. It is possible that both people are correct because they may have written the state $|\Psi\rangle$ in different bases. For example, Person 1 writes $\binom{a}{b} = \binom{\langle\phi_1|\psi\rangle}{\langle\phi_2|\psi\rangle}$ using basis $\{|\phi_1\rangle, |\phi_2\rangle\}$. Similarly, Person 2 writes $\binom{a'}{b'} = \binom{\langle\phi_1'|\psi\rangle}{\langle\phi_2'|\psi\rangle}$ using basis $\{|\phi_1'\rangle, |\phi_2'\rangle\}$.