Solution to Reflective Homework

(1) Person 1: Given the Hamiltonian $\hat{H}$ of the system and the initial wave function at time $t = 0$, we can find the wave function at all future times by solving the Time-Dependent Schroedinger Equation (TDSE).

This is correct from the statement of the TDSE which is the most fundamental equation of quantum mechanics.

Person 2: If the wave function can be determined with certainty at all future times, we should be able to determine the different physical observables such as position, momentum, energy etc. at all future times with certainty.

This statement is incorrect because, in general, the knowledge of the wave function can only give information about the probabilities of measuring different values for a physical observable. This is because the wavefunction in general will be a superposition of the eigenfunctions of the operator corresponding to a physical observable.

(2) "A wave function that is zero everywhere in the classically forbidden region cannot be a possible wave function for an electron interacting with a finite square well even if this wave function is single valued, continuous and smooth. This is because there should be a non-zero probability of finding the particle in the classically forbidden region if it is interacting with a finite square well."

This statement is incorrect. Although the stationary state wavefunctions for a finite square well have a non-zero probability of finding the particle in the classically forbidden region, we can take their linear superpositions to construct a wavefunction that is zero everywhere in the classically forbidden region.

(3) "For an infinite square well and a finite square well both between $0 \leq x \leq a$, some possible wave functions for the infinite square well can also be possible wave functions for the finite square well".

This statement is correct. For example, although the stationary state wavefunctions for a finite square well have a non-zero probability of finding the particle in the classically forbidden region, we can take their linear superpositions to construct a wavefunction that is zero everywhere in the classically forbidden region. We can also construct the same wavefunction by taking a suitable linear superposition of the stationary state wavefunctions for an infinite square well.

(4) The number of bound states will increase as the width of a finite square well is increased keeping the depth V_0 fixed because each of the energies will become lower and the spacing between the bound

states will decrease when the electrons become more de-localized in a wider well. One simple way to observe this is to consider how the energies of various energy levels and the spacing between the levels depends on the width a for an infinite square well. The expression for energies for the infinite square well is $E_n = n^2\pi^2\hbar^2/(2ma^2)$. It is clear from the expression that as a increases, each energy decreases and the separation between the two adjacent energy levels also decreases. Extending this analogy to the finite square well, we can conclude that more bound states will exist in a well that is deeper if V_0 is kept fixed.

(5) Two systems are identically prepared. In both systems, a free particle is in the <u>same</u> generic state $\phi(x)$ when momentum is measured for each. You measure the momentum in one of these systems and your friend measures the momentum in the other identically prepared system. You obtain a value P_0 for the momentum. But you cannot conclude much about your friend's measurement because depending upon the state $\phi(x)$, there can be a wide distribution in the possible values of momentum one can measure in identically prepared systems.

(6) A free particle is in a generic state $\phi(x)$ when you measure its momentum. After the first measurement, you wait for a long time before measuring its momentum again. The outcome for momentum measurement the second time will be the same as the first time. This is because the first measurement of momentum will collapse the system into a momentum eigenfunction. However, for a free particle, a momentum eigenfunction is also an energy eigenfunction (stationary state wavefunction) and if the wave function is a stationary state wavefunction, it remains in the stationary state unless a perturbation is applied. Therefore, if the momentum is measured again after a long time, the system is still in the same stationary state (which is also a momentum eigenstate in which the momentum has a definite value) and we will measure the same value of momentum as the first time. (Warning: For other potential energies, momentum eigenfunctions are NOT stationary state wavefunctions. Therefore, after the first momentum measurement, the wave function will collapse into a momentum eigenfunction which can be written as a linear superposition of stationary state wave functions. These different stationary states will have different time-dependent phase factors and the wave function will not remain a momentum eigenfunction at future times. In that case, the second momentum measurement could yield a different value from the first one).