Reflective Homework

(1) A Hilbert space consists of a set of vectors which satisfy certain properties (see textbook). A linear operator acts on a vector in a given Hilbert space and produces another vector. (Vectors are elements of a vector space. Linear operators are linear maps of a vector space onto itself.) For example, the Hilbert space for an electron confined in a one dimensional infinite square well is infinite dimensional. A state of the system $|\Psi\rangle$ is a vector in the Hilbert space. The Hamiltonian operator, position operator, momentum operator etc. are linear operators that act on state vectors in the Hilbert space. Eigenstates of any Hermitian operator form a complete set of vectors for the vector space (any vector in the vector space can be written as a linear superposition of a complete set of vectors).

(2) Statement 1: "In the formalism of quantum mechanics we learned, the wave function is represented by a Hermitian operator".

This statement is incorrect since physical observables are represented by Hermitian operators. The state of the system is a vector in the Hilbert space and in the position space, the state of the system is given by the wave function of the system.

Statement 2: "In quantum mechanics, the eigenstates of any Hermitian operator corresponding to a physical observable are called stationary states".

This statement is incorrect. Only the eigenstates of the Hamiltonian operator (corresponding to energy) are called stationary states. In a stationary state, the expectation value of all observables (that do not have an explicit time dependence) is time-independent.

(3) "In quantum mechanics, position and momentum are NOT "variables" but "operators". The measurement of position or momentum yields definite values of position or momentum which are *eigenstates* of the corresponding operator."

This statement is correct. Unlike classical mechanics, in quantum mechanics, position and momentum do not evolve in a deterministic manner. In quantum mechanics, corresponding to the observables position and momentum, we have corresponding position and momentum operators respectively that act on the state of the system (wave function). A measurement of position or momentum yields an eigenvalue of the position and momentum operator respectively.

(4) Three people have different opinions about whether "$|\alpha\rangle\langle\beta|$" represents an operator, a state vector, or a complex number within the Dirac formalism.

$|\alpha\rangle\langle\beta|$ is an operator which is an outer product of $|\alpha\rangle$ and $\langle\beta|$. One way to realize it is to act with it on a general state $|\Psi\rangle$ and it will yield another state $|\alpha\rangle\langle\beta|\Psi\rangle$ along the direction of the vector $|\alpha\rangle$.

(5) “If an electron interacting with a one-dimensional finite square well is initially in a state where the position has a definite value, the expectation value of position will be time-independent”.

This statement is incorrect. A position eigenstate is a state in which position has a definite value. It is a delta function in the position space. A position eigenstate is NOT an energy eigenstate (stationary state). If the system is in a position eigenstate (the wavefunction is a delta function in position space), it will not remain a delta function at future times. Since a position eigenstate is a linear superposition of many stationary states with different energies, each stationary state will evolve in time according to its own time-dependent phase factor. Therefore, the wave function will not be a delta function for future times and the expectation value of position will be time-dependent.