Reflective Homework

(1) Consider four normalized vectors representing the state of a quantum system in a two-dimensional Hilbert space (e.g., spin of an electron): $\binom{1}{0}$, $\binom{-1}{0}$, $\binom{i}{0}$ and $\binom{-i}{0}$.
Suppose you measure a physical observable Q. Is the probability of measuring different values of Q in each of these states different or same **in general**? Explain your reasoning.

(2) Consider four unnormalized vectors representing the state of a quantum system in a two dimensional Hilbert space (e.g., spin of an electron): $\binom{1}{1}$, $\binom{-1}{1}$, $\binom{-i}{1}$ and $\binom{i}{1}$.
Suppose you measure a physical observable Q. Is the probability of measuring different values of Q in each of these states different or same **in general**? Explain your reasoning.

(3) A person comments that x^n is linearly independent of 1, x, x^2,...$x^{(n-1)}$ because $x^n \neq a_0 + a_1 x + a_2 x^2 + ... + a_{n-1} x^{(n-1)}$ where $a_i, i = 0, 1, ..$ are any complex numbers. Another person claims that x^n is not linearly independent of $x^{(n-1)}$ because you can construct x^n from $x^{(n-1)}$ by using $x^n = x^{(n-1)} \times F(x)$ where $F(x) = x$ is a linear function. EXPLAIN why you agree or disagree with each person.

(4) Describe what basis vectors are and if they are unique for a given vector space. Illustrate whether they are unique or not with an example for a vector space of your choice.

(5) Two people describe the SAME state of a system $|\Psi\rangle$ in a two dimensional vector space by different column vectors. Person 1 writes column vector $\binom{a}{b}$ to describe $|\Psi\rangle$ and Person 2 writes column vector $\binom{a'}{b'}$ to describe $|\Psi\rangle$. In the expression each person writes to describe $|\Psi\rangle$, $a \neq a'$ and $b \neq b'$. Also, the first column vector <u>cannot</u> simply be written as the second column vector times a phase factor $e^{i\theta}$. Is it possible that both people are correct? You must justify your answer to get ANY credit.