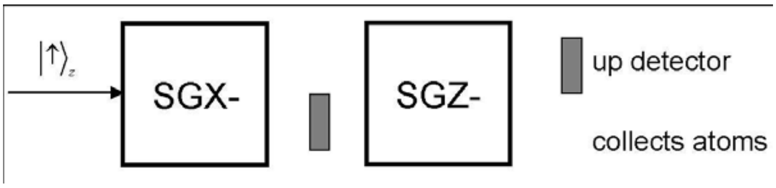
Homework for Stern-Gerlach Experiment

In the Stern-Gerlach tutorial, we have learned how to solve the following type of problem: Send silver atoms with initial spin state $\left|\uparrow\right\rangle_z$ through SGX- and SGZ-. One detector is placed in the lower channel after SGX, and another detector is placed in the upper channel after SGZ-. We learned how to determine the probability of each detector clicking and the state we prepared after each SGA.



Let's discuss what changes take place if we only place a detector after the second SGA or if we do not place any detector after either of the SGAs.

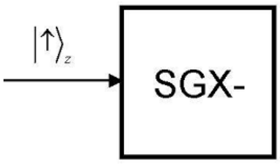
1. As shown in the figure below, suppose we send silver atoms in the spin state $\left|\uparrow\right\rangle_z$ at time t=0 through a SGX- without a detector placed after it. What is the spin wavefunction $\left|\chi(T)\right\rangle$ of an atom right after it passes through SGX-? Assume that the magnetic field is $\vec{B} = C_0 x\hat{i}$ (C_0 is a constant) and the atoms stay in the SGX- for a time $t = T$.

A. $\left|\chi(T)\right\rangle = \frac{1}{\sqrt{2}}\phi_+\left|\uparrow\right\rangle_z + \frac{1}{\sqrt{2}}\phi_-\left|\downarrow\right\rangle_z$, where $\phi_\pm = e^{\pm iC_0\gamma\cdot zT/2}$

B. $\left|\chi(T)\right\rangle = \frac{1}{\sqrt{2}}\phi_+\left|\uparrow\right\rangle_x + \frac{1}{\sqrt{2}}\phi_-\left|\downarrow\right\rangle_x$, where $\phi_\pm = e^{\pm iC_0\gamma\cdot xT/2}$

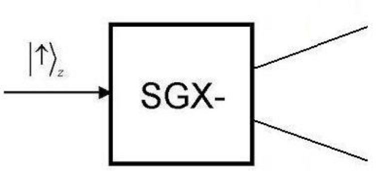
C. $\left|\chi(T)\right\rangle = \left|\uparrow\right\rangle_z$

D. $\left|\chi(T)\right\rangle = \frac{1}{\sqrt{2}}\left|\uparrow\right\rangle_x + \frac{1}{\sqrt{2}}\left|\downarrow\right\rangle_x$

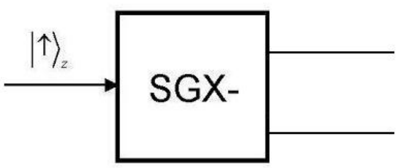


2. In the previous problem (question 1), the spin up component $|\uparrow\rangle_x$ gets coupled with a spatial term involving momentum $p_x = \frac{C_0 \gamma \cdot T}{2}$ (T is the total time that the atoms stay in the SGX-) and the spin down component $|\downarrow\rangle_x$ is coupled with a spatial term involving momentum $-p_x$, so we have $\Psi = \frac{1}{\sqrt{2}} e^{i p_x \cdot x/\hbar} |\uparrow\rangle_x + \frac{1}{\sqrt{2}} e^{-i p_x \cdot x/\hbar} |\downarrow\rangle_x$. <u>Choose all of the following statements that are correct.</u>

1) As illustrated in the schematic diagram below, the spin up component $|\uparrow\rangle_x$ and spin down component $|\downarrow\rangle_x$ will get spatially separated with the separation increasing with time after the SGX-, because they are coupled with phase factors involving momentum components in opposite directions.



2) As illustrated in the schematic diagram below, the spin up component $|\uparrow\rangle_x$ and the spin down component $|\downarrow\rangle_x$ will appear as two parallel spatially separated beams after the SGX-, because they get spatially separated inside the SGX- but then become parallel right after the SGX-.



3) Since the Hamiltonian for the SGA couples the momentum to the spin degree of freedom, the clicking of an appropriately placed detector (in the upper or lower channel) after the SGX- will inform us about the spin state of each atom.

A. only 1
B. only 2
C. 1 and 3
D. 2 and 3
E. None of the above

3. Now let's use (1) and (2) to represent the spatial part of the wavefunction in *Figure 1*, e.g., $\frac{1}{\sqrt{2}}|\uparrow\rangle_x \cdot (1) + \frac{1}{\sqrt{2}}|\downarrow\rangle_x \cdot (2)$. We place a detector in the upper channel as shown in *figure 2* and send the atoms one by one. When the detector does not click, what is the wavefunction of the atom? (Do not forget the spatial parts (1) and (2).)

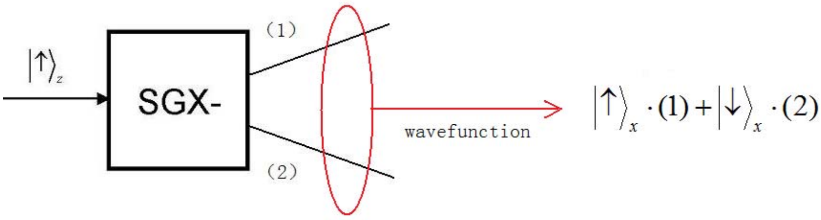


Figure 1

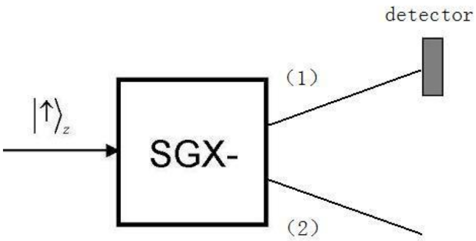
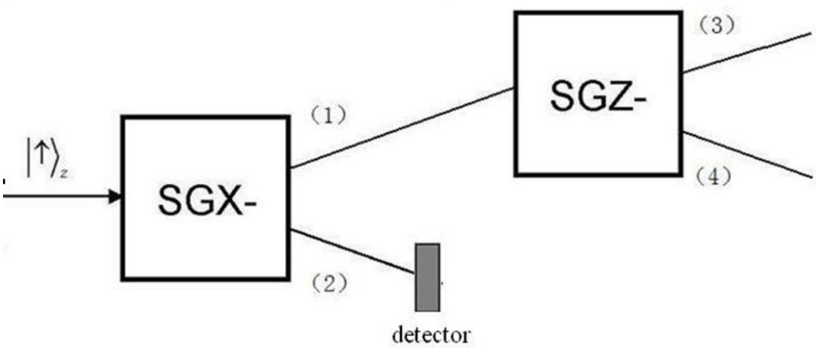
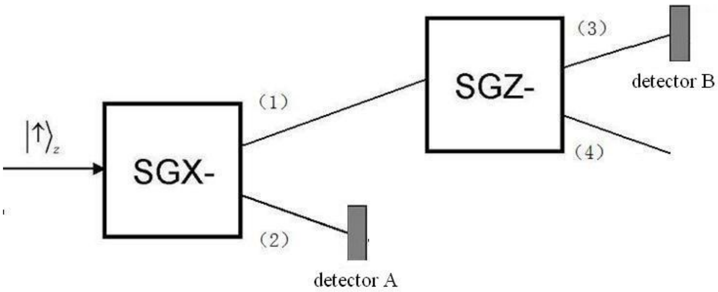


Figure 2

4. As illustrated in the schematic diagram below, a SGZ- is set in the upper channel after SGX-. If we place a detector in the lower channel after SGX- (position 2) and send a silver atom in the state as shown, what is the probability that the detector would click? Explain.



5. In the previous question (*question 4*), if we place one detector (detector A) in the lower channel after SGX- (position 2) and another detector (detector B) in the upper channel after SGZ-, what is the probability that detector B would click? Explain by explicitly writing down the spin wavefunction right before the detector B after passing through the SGZ-. You can write the spatial parts of the wavefunction in terms of (3) and (4) as illustrated in the figure.



6. In the previous problem (*question 5*), if we do not use any detector (see Figure 3 below), what would be the spin wavefunction of the atom after it passed through SGZ-? Here (3) and (4) correspond to the spatial parts of the wavefunction in the upper and lower channels after SGZ-.

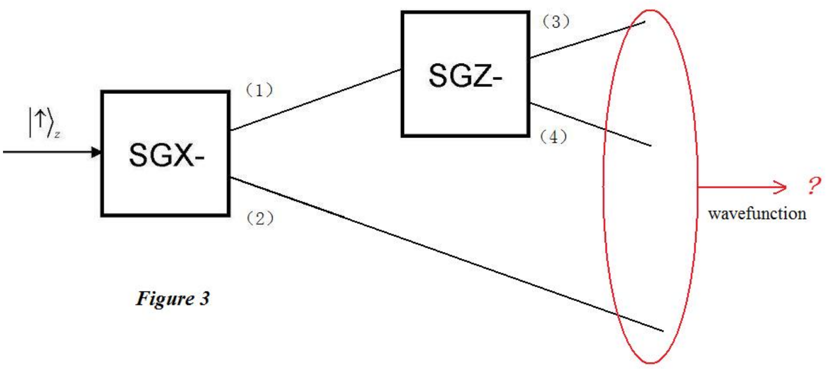


Figure 3

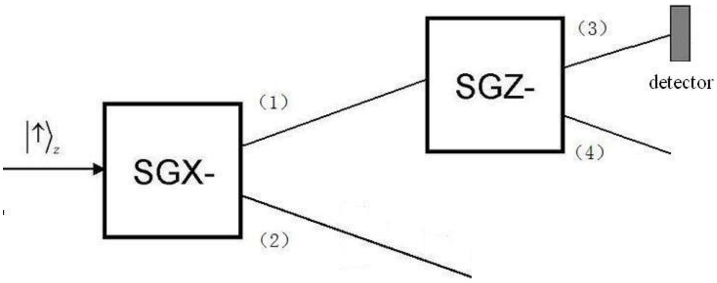
A. $\frac{1}{\sqrt{2}}|\uparrow\rangle_x \cdot (1) + \frac{1}{\sqrt{2}}|\downarrow\rangle_x \cdot (2)$

B. $\frac{1}{\sqrt{2}}|\uparrow\rangle_z \cdot (3) + \frac{1}{\sqrt{2}}|\downarrow\rangle_z \cdot (4)$

C. $\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|\uparrow\rangle_z \cdot (3) + \frac{1}{\sqrt{2}}|\downarrow\rangle_z \cdot (4)\right) + \frac{1}{\sqrt{2}}|\downarrow\rangle_x \cdot (2)$

D. $\frac{1}{\sqrt{3}}|\uparrow\rangle_z \cdot (3) + \frac{1}{\sqrt{3}}|\downarrow\rangle_z \cdot (4) + \frac{1}{\sqrt{3}}|\downarrow\rangle_x \cdot (2)$

7. In the previous question (*question 6*), if we place a single detector in the upper channel after SGZ- (see the Figure below) and send a silver atom in the spin state shown, what is the probability that the detector would click? Explain. Use your answer in the question 6 to justify your answer.



8. Is your result to *question 5* the same as your answer to *question 7*? Does placing a detector in the lower channel after SGX- (as in question 5) make it easier to write down the spin wavefunction and calculate the probability of clicking of the upper detector after SGZ- when an atom is sent? Explain.

9. In *question 7*, if the detector does not click, what is the spin wavefunction of the atom?