**Double-slit Experiment with Single Particles: Tutorial**

Relativistic effects are ignored in this tutorial.

For any constant, e.g., the mass of an electron or muon or Planck's constant, use the following values:

* 1 eV = 1.6 ×10-19 J
* keV = kilo electron volt = 103 eV, meV = milli electron volt = 10-3 eV
* 1 mm = 10-3 m, 1 µm = 10-6 m, 1 nm = 10-9 m, 1 pm = 10-12 m

* Planck's constant = *h* = 6.6 × 10-34 Js
* magnitude of elementary charge (on an electron or proton) = *e* = 1.6 × 10-19 C
* speed of light = *c* = 3 × 108 m/s
* mass of electron = 9.1 × 10-31 kg
* mass of neutron = mass of proton = 1.67 × 10-27 kg
* mass of muon = 1.9 × 10-28 kg
* mass of helium atom = 6.68 × 10-27 kg
* mass of sodium atom = 3.8 × 10-26 kg

Terms used in this tutorial and their meanings:

* $Ψ(x)$ = Probability density amplitude (also referred to as wave function)
* $\left|Ψ(x)\right|^{2}$ = Probability density
* $\left|Ψ(x)\right|^{2}dx$ = Probability of finding the particle described by the wave function $Ψ(x)$ between position *x* and *x* + *dx*

Throughout this tutorial, assume that all wave functions are normalized. Also, in general, the wave functions are complex (i.e., not real) functions.

**Double slit Experiment with Single Particles**

**Acknowledgment: The simulation used in this tutorial was developed by Klaus Muthsam.**

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**This tutorial is designed to help you learn:**

* the importance of adding the probability amplitude (and not the probability density) for understanding interference of particle-waves in quantum mechanics.
* the relation between having information about which slit a “particle” went through (“which-path” information) and the loss of the interference pattern.

It is recommended that you work through this tutorial with a blank sheet of paper and pen so that you can make calculations and write down important ideas you learn as you go along.

**Double-Slit Experiment: Set-up**

Open the simulation of the double-slit experiment. Select English from the language menu and click ok. You will see the experimental set-up for the “double-slit” experiment.

* The red box is the source which can emit different types of particles.
* The black screen has two slits on it. It comes right after the red source box.
* There is a lamp (bulb) in between the double slit and the white screen.
* There is a white screen after the lamp. When particles passing through the double-slit one at a time hit the screen, their interaction with the atoms in the screen causes one red “spot” on the screen for each particle.

**Double-slit experiment and issues related to the foundations of quantum mechanics**

1. Consider the following statements about the electrons passing through the double-slit one at a time:

1. While propagating from the source to the screen, each electron can be considered as a wave delocalized in space which can pass through both slits at the same time.
2. Each electron can interfere with itself.
3. Each electron can pass through only one slit with a 50% probability of passing through each slit.

Choose all of the above statements that are true.

(a) I only

(b) II only

(c) III only

(d) I and II only

(e) None of the above

Explain your reasoning.

2. Consider the following statements about the interference pattern produced by the electrons passing through the double-slit one at a time (Assume that when an electron arrives at the fluorescent screen, a flash is produced):

1. We cannot predict ahead of time at which point on the screen an electron will arrive.
2. The interaction of each electron with the atoms in the screen collapses the wave function of the electron and makes it localized in position.
3. Each flash of light observed when an electron arrives at the screen is equivalent to a measurement of the electron's position.

Choose all of the above statements that are true.

(a) I and II only

(b) I and III only

(c) II and III only

(d) All of the above

Explain your reasoning.

**“Which-path” information and the loss of the interference fringes**

The double slit experiment with single electrons (one electron at a time) is pictured below:



Consider the two possible situations that can arise in the double-slit experiment with single electrons:

**Case 1:** For each electron, we have information about the path taken by the electron because at the slits, it is localized in position at one slit or the other.

**Case 2:** For each electron, we do not have information about the path taken by the electron because at the slits, it is delocalized in position across both slits.

We now want to understand how these two cases differ in terms of what is observed on the screen using a simplified quantum mechanical formulation.

* Assume that the electrons travel the same distance from the source to the slits.

**Case 1**

* If slit 2 is closed and an electron goes through slit 1, the wave function (probability density amplitude) of that electron at a point *x* on the screen (right before the interaction with the screen, which corresponds to a measurement of position as the fluorescent screen absorbs the electron and makes it extremely localized in position at a specific point on the screen) is $Ψ\_{1}(x)$.
* If instead, slit 1 is closed and the electron goes through slit 2, the wave function (probability density amplitude) of the electron at a point *x* on the screen is $Ψ\_{2}(x)$.

**Case 2**

* If we do not know which slit an electron went through because the electron is delocalized across both slits, the wave function (probability density amplitude) of that electron at a point *x* on the screen is $Ψ\_{1}\left(x\right)+Ψ\_{2}(x)$.
	+ This is equivalent to the description that “the electron went through both slits simultaneously”.
* For a given point on the screen, the path length from one slit is *L*1 and the path length from the other is *L*2. In general, *L*1*­* and *L*2 are different (they are only equal for one point on the screen – the central point equidistant from both slits).
	+ The phases of the complex wave functions, $Ψ\_{1}(x)$ and $Ψ\_{2}(x)$, at a given point *x* on the screen are related to the path lengths: $ϕ\_{i}=\frac{2π}{λ}L\_{i}$ (i = 1,2) where *λ* is the wavelength of the electron.
	+ The phases of the two complex wave functions are therefore different in general.
* Therefore, in complex notation (magnitude and phase form) the two wave functions at a given point *x* on the screen will look like $Ψ\_{1}(x)= \left|Ψ\_{1}(x)\right|e^{iϕ\_{1}}$ and $Ψ\_{2}(x)= \left|Ψ\_{2}(x)\right|e^{iϕ\_{2}}$ where the phases $ϕ\_{1}$ and $ϕ\_{2}$ are in general different.
	+ The phases $ϕ\_{1}$ and $ϕ\_{2}$ are functions of *x* because they depend on the path lengths *L*1 and *L*2 which depend on the location of point *x* on the screen.

Now, let’s consider one electron emitted by the source arriving at point *x* on the screen in each of the two cases:

**Case 1: We have information about which slit the electron went through**

* If slit 2 is closed and the electron goes through slit 1, the wave function of the electron at a point *x* on the screen (right before the interaction with the screen that makes the electron wave function localized in position) is $Ψ\_{1}(x)= \left|Ψ\_{1}(x)\right|e^{iϕ\_{1}}$.
* If slit 1 is closed and the electron goes through slit 2, the wave function of the electron at a point *x* on the screen (right before the interaction with the screen that makes the electron wave function localized in position) is $Ψ\_{2}(x)= \left|Ψ\_{2}(x)\right|e^{iϕ\_{2}}$.

**Case** **2: We do not have information about which slit the electron went through**

* The electron goes through both slits at the same time
* The wave function of the electron at a point *x* on the screen (right before the collapse of the wave function due to the interaction with the screen) is a linear superposition of the wave functions $Ψ\_{1}(x)$ and $Ψ\_{2}(x)$:
	+ $Ψ(x)=\frac{1}{\sqrt{2}}\left(Ψ\_{1}(x)+Ψ\_{2}(x)\right)=\frac{1}{\sqrt{2}}\left(\left|Ψ\_{1}(x)\right|e^{iϕ\_{1}}+ \left|Ψ\_{2}(x)\right|e^{iϕ\_{2}}\right)$.
	+ $\frac{1}{\sqrt{2}}$ in $Ψ(x)$ ensures that the wave function $Ψ(x)$ is normalized.

In order to determine what pattern forms on the screen after a large number *N* ofelectrons have arrived at the screen, we need to calculate the expected number density of electrons at each location on the screen. We do this by summing over the probability densities of each of the *N* electrons that arrive at a particular point *x* on the screen. To obtain the probability density for one electron, one must take the absolute square of the wave function (probability amplitude) for that electron to arrive at a particular point on the screen. The wave function (probability amplitude) of an electron can be $Ψ\_{1}(x)$ or $Ψ\_{2}(x)$ if we know which slit the electron went through, or $\frac{1}{\sqrt{2}}\left(Ψ\_{1}(x)+Ψ\_{2}(x)\right)$ if we do not know which slit the electron went through (i.e., the electron went through both slits simultaneously).

**Case 1: We have information about which slit the electrons went through (i.e., for each electron, we know whether it went through slit 1 or slit 2)**

3. Calculate the expected number density of electrons at a point *x* on the screen after *N* electrons arrived at the screen. You can assume that the wave function of some electrons (say, *N*1 – which is roughly *N*/2) that went through slit 1 and arrived at point *x* on the screen is $Ψ\_{1}(x)$ and the wave function of the rest of the electrons (say, *N*2) that went through slit 2 and arrived at point *x* on the screen is $Ψ\_{2}(x)$ (of course, *N*1 + *N*2 = *N*).

**Case** **2: We do not have information about which slit the electrons went through (each electron went through both slits)**

4. You will follow a few steps to calculate the expected number density of electrons at a point *x* on the screen, after *N* electrons arrived at the screen for case 2. Keep in mind that in this case, the wave function of each electron at the screen is the same, namely, the linear superposition written earlier, $Ψ(x)=\frac{1}{\sqrt{2}}\left(Ψ\_{1}(x)+Ψ\_{2}(x)\right)=\frac{1}{\sqrt{2}}\left(\left|Ψ\_{1}(x)\right|e^{iϕ\_{1}}+ \left|Ψ\_{2}(x)\right|e^{iϕ\_{2}}\right)$.

(i) Find $\left|Ψ(x)\right|^{2}=Ψ(x)∙Ψ^{\*}(x)$ (probability density for one electron) in terms of $\left|Ψ\_{1}(x)\right|$, $\left|Ψ\_{2}(x)\right|$, $ϕ\_{1}$ and $ϕ\_{2}$.

(ii) If you have not done so, combine the phase terms in the exponent. Then, write$ ϕ\_{1}-ϕ\_{2}=Δϕ$ and find $\left|Ψ\right|^{2}$ in terms of $\left|Ψ\_{1}\right|$, $\left|Ψ\_{2}\right|$ and $Δϕ$.

(iii) Use the identity $e^{iα}=\cos(α)+i\sin(α)$ and take advantage of the fact that sine is an odd function and cosine is an even function to simplify the expression.

(iv) What is the expected number density of electrons at a point *x* on the screen in case 2 in which we do not have information about which slit the electrons went through?

**Summary: Expressions for the expected number density of electrons in Cases 1 and 2:**

**Case 1:** We have “which-path” information (we know whether the electrons went through slit 1 or slit 2)

* Expected number density of electrons $=N\_{1}∙\left|Ψ\_{1}(x)\right|^{2}+N\_{2}∙\left|Ψ\_{2}(x)\right|^{2}$

**Case 2:** We do not have “which-path” information

* Expected number density of electrons $= \frac{N}{2}∙\left(\left|Ψ\_{1}(x)\right|^{2}+\left|Ψ\_{2}(x)\right|^{2}+2\left|Ψ\_{1}(x)\right|∙\left|Ψ\_{2}(x)\right|∙\cos(Δϕ)\right)$

5. Based on the two expressions above, in which one of these two cases do you expect to observe an interference pattern on the screen? Explain.

6. Consider the following statements about the double-slit experiment:

1. If the cross term ($2\left|Ψ\_{1}(x)\right|∙\left|Ψ\_{2}(x)\right|∙\cos(Δϕ)$) in the expression for the expected number density of electrons is negligible in a given situation, interference effects will be negligible.
2. If we obtain “which-path” information, i.e., information about which slit the electron went through, the cross term in the expression for the expected number density of electrons vanishes.
3. If we first square the wave function from each slit and then add them to obtain the total probability density for a single electron, i.e., $\left|Ψ(x)\right|^{2}=\left|Ψ\_{1}(x)\right|^{2}+\left|Ψ\_{2}(x)\right|^{2}$, and then sum over all electrons to obtain the expected number density of electrons at each point *x* on the screen, we would conclude that there are no interference effects.

Choose all of the above statements that are true.

(a) I and II only

(b) I and III only

(c) II and III only

(d) I, II and III

Explain your reasoning.

**Summary: “Which-path” information: probability density and expected number density of electrons at a point *x* on the screen**

* To determine what pattern forms after *N* electrons have arrived at the screen, we must obtain the **expected number density of electrons at each location on the screen** which we calculate by summing over the probability density of each electron. We do this for each of the two cases.

**Case 1: We have “which-path” information (we know which slit the electrons went through)**

* *N*1 electrons went through slit 1 and *N*2 electrons went through slit 2 (*N*1 and *N*2 are roughly *N*/2 and they sum to *N*, the total number of electrons)
* The wave function of the *N*1 electrons at a point *x* on the screen is $Ψ\_{1}(x)$ and wave function of the *N*2 electrons at a point *x* on the screen is $Ψ\_{2}(x)$.
	+ Expected number density of electrons:

$$N\_{1}∙\left|Ψ\_{1}\left(x\right)\right|^{2}+ N\_{2}∙\left|Ψ\_{2}(x)\right|^{2}$$

* + There is no oscillating term ($2\left|Ψ\_{1}(x)\right|∙\left|Ψ\_{2}(x)\right|∙\cos(Δϕ)$) in this case and therefore there will be no interference pattern on the screen.

**Case 2: We do not have information about which slit the electrons went through**

* The wave function of each of the *N* electrons is in a linear superposition of the wave functions from each slit: $Ψ(x)=\frac{1}{\sqrt{2}}\left(\left|Ψ\_{1}(x)\right|e^{iϕ\_{1}}+\left|Ψ\_{2}(x)\right|e^{iϕ\_{2}}\right)$.
* Each electron goes through both slits at the same time.
* Each electron interferes with itself and an interference pattern is observed on the screen after a large number of electrons have arrived at the screen.
	+ Expected number density of electrons at a point *x* on the screen:

$$\frac{N}{2}∙\left(\left|Ψ\_{1}(x)\right|^{2}+\left|Ψ\_{2}(x)\right|^{2}+2\left|Ψ\_{1}(x)\right|∙\left|Ψ\_{2}(x)\right|∙\cos(Δϕ)\right)$$

* + The last term, which includes the oscillating cosine function, is responsible for the interference pattern.
	+ Note that it is the *phase difference*, $Δϕ=ϕ\_{1}-ϕ\_{2}=\frac{2π}{λ}\left(L\_{1}-L\_{2}\right)$ between $Ψ\_{1}(x)$ and $Ψ\_{2}(x)$ that is responsible for the interference pattern observed at the screen (i.e., the absolute phase is not important, only the difference between the phases of $Ψ\_{1}(x)$ and $Ψ\_{2}(x)$ is important).

**Loss of the interference pattern due to obtaining “which-path” information as a result of electron-photon scattering**

Consider the modified double slit experiment pictured below:

This is the usual setup of the double slit experiment with the addition of a lamp between the slits and the screen which emits monochromatic light (all photons emitted by the lamp have the same wavelength). This picture is taken from a simulation you will use to learn how electron-photon scattering in the region between the slits can provide “which-path” information and destroy the interference pattern. In the simulation, the lamp is placed slightly off to one side. However, this is just a schematic diagram of the lamp and it is assumed that a photon emitted by the lamp has equal probability of being scattered by an electron from either slit. On its way from the source to the screen, an electron can scatter off a photon emitted by the lamp and in the scattering process, the electron becomes localized within a region of similar length scale as the wavelength of the photon. A major assumption of this thought experiment is that scattering between an electron and a photon can only occur at the slits.

In addition, in the simulation, the intensity of the lamp can be varied from 1% to 100%. The way the simulation interprets an intensity of *X*% is that *X*% of the electrons scatter off photons. For example, if the intensity of the lamp in the simulation is 50%, that means that half of the electrons scatter off photons and half go from the source to the screen without scattering off photons.

Consider the following conversation between Pria, Mira and Nancy about why an important consideration in the loss of the interference fringes is the comparison of the slit separation with the wavelength of the photons emitted by the lamp:

* Pria: I think that we will always have “which-path” information regardless of the wavelength of the photons emitted by the lamp as long as the lamp has high intensity. If the lamp has high intensity, virtually every electron will scatter off one photon. Therefore, we will be able to determine where each electron scattered from (which slit it went through) based upon the information about the scattered photon.
* Mira: I disagree with your conclusion. If the photon had very large wavelength compared to the distance between the slits, it would not matter if an electron scatters off a photon because diffraction will limit our ability to resolve length scales smaller than the wavelength of the photon (and therefore not provide information about which slit the electron went through). For example, due to diffraction, one cannot use an optical microscope to examine viruses because their size is smaller than the shortest wavelength of visible light.
* Nancy: I agree with Mira. If you are not able to resolve two things by using photons of a particular wavelength, this means that from the point of view of a photon, those two things are difficult to separate. In this context, if we are using photons with a wavelength larger than the distance between the slits, from the point of view of a photon, those two slits overlap and could be indistinguishable. If instead, the wavelength of the photon is smaller than the distance between the slits, a photon which scatters off an electron at one slit or another can provide information about which slit the interference occurred.

Do you agree with Pria and/or Mira and Nancy? Explain.

7. Assume that you performed a double slit experiment with single electrons and saw an interference pattern on the screen. Suppose that now you place a lamp between the double-slit and the screen. The lamp has a high intensity and each electron interacts with (scatters off) one photon.

(i) Which one of the following statements is correct if the wavelength of the photons emitted by the lamp is significantly less than the distance between the slits?

(a) The interference pattern will go away.

(b) The interference pattern essentially remains unchanged.

(c) The interference pattern is still visible, however, it is harder to discern because of reduced contrast.

(d) The interference pattern becomes easier to discern because of increased contrast.

(ii) Explain your reasoning for your answer in (i).

You will now use the simulation to check your answer to question 7.

* Go to the source menu in the simulation and select electrons with energy 200 keV.
* Go to the aperture menu and select a slit width of 100 nm and a slit distance of 2000 nm.
* Turn on the source (do NOT turn on the light bulb yet), hold down the speed button until the maximum number of electrons is used and adjust the zoom until you see 2-5 interference fringes (100,000x should be optimal).
* Before you turn on the lamp, take a close look at the interference pattern on the screen.
* Turn on the lamp and set the intensity to the maximum value, 100% (which means that all the electrons will scatter off one photon) and the wavelength to the minimum possible value (380 nm, which is significantly less than the distance between the slits).

8. What happened to the interference pattern after the lamp was turned on? Is this observation consistent with your answer to question 7? If it is not, reconcile the difference between your prediction and observation.

So far we only considered the case in which the lamp has high intensity which can be interpreted to mean that due to the very large number of photons emitted by the lamp each second, the probability that an electron scatters off a photon is 100%. In other words, every electron scatters off a photon. However, the intensity of the lamp can be reduced to an intermediate value (between 1% and 99%), in which case some electrons do not scatter off photons while others do.

9. (i) Consider a case in which the lamp has intermediate intensity such that half of the electrons do not scatter off photons. Which one of the following statements is correct if the wavelength of the photons emitted by the lamp is significantly less than the distance between the slits?

(a) The interference pattern will go away.

(b) The interference pattern essentially remains unchanged.

(c) The interference pattern is still visible, however, it is harder to discern because of reduced contrast.

 (d) The interference pattern becomes easier to discern because of increased contrast.

 (ii) Explain your reasoning for your answer in (i).

You will now use the simulation to check your answer to question 9.

* Go back to the previous simulation (electrons of energy 200 keV, slit width 100 nm and slit distance 2000 nm).
* When you turned on the lamp which emitted photons of wavelength 380 nm and set the intensity of the lamp to 100% you observed that the interference pattern went away. This is because the wavelength of the photons was significantly smaller than the slit distance and due to the high intensity of the lamp, each electron scattered off one photon at one of the slits. This scattering provided “which-path” information for each electron and destroyed the interference pattern.
* Now lower the intensity of the lamp by 20% five times in a row until you reach the minimum (1%). Every time, press the speed button until the maximum number of electrons has been used in the simulation and look closely at the pattern on the screen.

10. What happened to the interference pattern as you lowered the intensity? Is this observation consistent with your answer to question 9? If it is not, reconcile the difference between your prediction and observation.

**Summary: Wavelength of photons emitted by the lamp is significantly smaller than the slit distance**

* If the wavelength of the photons emitted by the monochromatic lamp is significantly smaller than the slit distance, every time a photon scatters off an electron, we have “which-path” information (information about which slit the electron went through).
	+ The scattering process between an electron and a photon localizes the electron within a region of length scale comparable to the wavelength of the photon.
	+ Since scattering is assumed to occur only at the slits and the wavelength of the photons is significantly smaller than the slit distance, each scattering process localizes an electron in a region much smaller than the distance between the slits.
	+ We therefore obtain “which-path” information for each electron because while the electron is at the slits, scattering with the photon localizes the electron in a region small enough to be able to say with certainty whether the electron is at one slit or at the other.
* When we have “which-path” information, the wave function of the electrons scattered by the photons from the lamp is no longer in a linear superposition of the wave functions from the two slits
	+ The scattered electrons will not produce an interference pattern because they follow the probability density distribution predicted for the case in which we have information about the slit each electron went through.
		- This probability density does not have an oscillating term (involving $\cos(Δϕ)$) discussed earlier.
* If the lamp has intensity such that each electron scatters off one photon (100% intensity in the simulation), there will be no interference pattern on the screen because the scattering of an electron off a photon yields information about which slit the electron went through.
* It is important to emphasize that one does not need to measure (via some apparatus) the direction in which each photon scattered off an electron and use this information to experimentally obtain “which-path” information. The mere fact that it is theoretically possible to obtain “which-path” information is enough to destroy the interference pattern.
* If the intensity of the lamp is such that some electrons scatter off photons and some do not (between 1% and 99% in the simulation)
	+ Some electrons will not scatter off photons and will produce an interference pattern.
	+ Other electrons do scatter and do not contribute to the interference pattern.
	+ The lower the intensity of the lamp, the higher the contrast of the interference pattern (the electrons that scatter off photons lower the contrast because they do not follow the probability distribution predicted for interference and therefore do not contribute to the interference pattern).

Now consider the following situations in which the wavelength of the photons emitted by the lamp is initially significantly larger than the distance between the slits, and then the wavelength of the photons is gradually reduced.

11. Assume that you performed a double slit experiment with single electrons and saw an interference pattern on the screen. Suppose that now you place a lamp between the double-slit and the screen. The lamp has high intensity and each electron interacts with (scatters off) one photon.

(i) Which one of the following statements is correct if the wavelength of the photons emitted by the lamp is significantly larger (say 2, 3 times larger) than the distance between the slits?

(a) The interference pattern will go away.

(b) The interference pattern essentially remains unchanged.

(c) The interference pattern is still visible, however, it is harder to discern because of reduced contrast.

 (d) The interference pattern becomes easier to discern because of increased contrast.

(ii) Explain your reasoning to your answer in (i).

You will now use the simulation to check your answer to question 11.

* Go to the source menu in the simulation and select electrons with energy 50 keV.
* Go to the aperture menu and select a slit width of 100 nm and a slit distance of 300 nm.
* Turn on the source (do NOT turn on the light bulb yet), hold down the speed button until the maximum number of electrons is used and adjust the zoom until you see 2-5 interference fringes (10,000x should be optimal).
* Before you turn on the lamp, take a close look at the interference pattern on the screen.
* Turn on the lamp and set the intensity to the maximum value, 100% (which means that all the electrons will scatter off at least one photon).
* Set the wavelength to the maximum possible value, 780 nm. Turn on the source and hold the speed button down until the maximum number of electrons has been used.
	+ Note that in this case, the wavelength of the light from the lamp is 2.6 times larger than the distance between the slits (i.e., significantly larger).

12. What happened to the interference pattern when you turned the lamp on? Was it significantly different from the interference pattern obtained for the case when the lamp was off? Does your observation agree with your answer to question 11? If it does not, reconcile the difference between your prediction and observation.

13. Assume you performed a double slit experiment with single electrons and observed an interference pattern. Then you set the wavelength of the photons emitted by the lamp to be significantly larger than the distance between the slits, turned the lamp on and repeated the experiment. You observed that the interference pattern remained the same. Suppose now you gradually reduce the wavelength of the photons emitted by the lamp and repeat the experiment multiple times until the wavelength of the photons becomes close to the distance between the slits.

(i) What would happen to the interference pattern?

(a) It will go away.

(b) It essentially remains unchanged.

(c) The interference pattern gradually becomes harder to discern.

(d) The interference pattern gradually becomes easier to discern.

 (ii) Explain your reasoning for your answer in (i).

* You will now use the simulation to check your answer to question 13.
	+ In order to ensure that you do not observe differences in the interference patterns due to the total number of electrons used by the simulation, every time you modify a parameter, hold down the speed button until the maximum number of electrons is used.
* Go back to the previous simulation (electrons of energy 50keV, slit width 100 nm and slit distance 300 nm, wavelength of photons emitted by the lamp 780 nm, lamp intensity 100%).
	+ Note that in this case the wavelength of the photons (780 nm) is 2.6 times larger (i.e., significantly larger) than the distance between the slits (300 nm).
	+ Turn on the source and hold down the speed button until the maximum number of electrons is used.
	+ Take a close look at the interference pattern.
* Now, reduce the wavelength of light by 100 nm and repeat the usual steps to observe an interference pattern on the screen. Is it significantly different from the previous interference pattern?
	+ Repeat this step three more times until you reach the smallest possible value (380 nm). You should observe five interference patterns (for photon wavelengths of 780, 680, 580, 480 and 380 nm).

14. What did you observe happen to the interference pattern on the screen as the wavelength of the photons emitted by the lamp decreased and became closer to the distance between the slits? Does this observation agree with your answer to question 13? If it does not agree, reconcile the difference between your prediction and observation.

**Summary: The wavelength of the photons emitted by the lamp is larger than the slit distance**

* If the wavelength of the photons emitted by the lamp is larger than the distance between the slits, diffraction will limit our ability to resolve which slit the electrons went through from the knowledge of the direction of the scattered photon.
	+ Each scattering event between an electron and a photon localizes the electron in a region of length scale comparable to the wavelength of the photon.
	+ But the wavelength of the photon is larger than the distance between the slits and therefore, while the electron is at the slits, it is not localized in a region small enough to be able to tell with certainty that it is at one slit or at another.
		- In other words, from the point of view of the photon, the two slits overlap (they are not clearly distinguishable from each other).
	+ During each scattering event that occurs at the slits, there is a level of uncertainty about whether it happens at slit 1 or at slit 2, and this level of uncertainty is related to the wavelength of the photon (larger wavelength 🡪 more uncertainty).
		- If the wavelength of the photons is large compared to the distance between the slits, the probability that we have “which-path” information is small.
	+ Therefore, for the case in which the wavelength of the photons emitted by the lamp is large compared to the distance between the slits, the interference pattern will remain essentially unchanged (compared to when the lamp is turned off).
* If the wavelength of the photons is then decreased, the probability of obtaining “which-path” information increases, and we will have “which-path” information for more electrons.
	+ The larger the number of electrons for which we have “which-path” information, the more difficult it will be to discern the interference pattern because of reduced contrast.
	+ Once the wavelength of the photons emitted by the lamp becomes comparable to (or shorter than) the distance between the slits, we have “which-path” information for most of the electrons.
		- Therefore, the interference pattern becomes very faded, almost impossible to discern from the background of electrons that do not exhibit interference.

In the following three questions, assume that when the lamp is turned off, there is a clearly visible interference pattern on the screen.

15. Suppose that the wavelength of the photons emitted by the lamp is equal to 1.5*d* where *d* is the distance between the slits. Explain what happens to “which-path” information (the probability of obtaining it) as the intensity of the lamp is decreased from 100% to 0% and what the corresponding effect is on the observed interference pattern. Consider intermediate situations as well (i.e., do not consider the extreme cases of 100% and 0% only).

16. Suppose that the wavelength of the photons emitted by the lamp is equal to 3*d*. Explain what happens to “which-path” information (the probability of obtaining it) as the intensity of the lamp is decreased from 100% to 0% and what the corresponding effect is on the observed interference pattern. Consider intermediate situations as well (i.e., do not consider the extreme cases of 100% and 0% only).

17. Suppose that the lamp intensity is 100%. Explain what happens to “which-path” information (the probability of obtaining it) as the wavelength of the photons emitted by the lamp is decreased from 3*d* to somewhat less than *d* (e.g., *d*/2) and what the corresponding effect is on the observed interference pattern. Consider intermediate situations as well (i.e., do not consider the extreme cases of 3*d* and less than *d* only).

18. Summarize what you learned by completing the following table.

In each box below, place a number from 1 to 4 that corresponds to the contrast of the interference pattern: 4 corresponds to the maximum (or very close to maximum) contrast (as in the case when the light bulb is turned off) and 1 corresponds to the minimum (or very close to the minimum) when the interference pattern is either completely gone, or nearly impossible to discern from the random background due to reduced contrast. In all of these cases, assume that there is a clearly visible interference pattern when the light bulb is turned off. In the table, *d* refers to the distance between the slits

|  |  |  |  |
| --- | --- | --- | --- |
|  | Lamp intensity = 100% | Lamp intensity = 60% | Lamp intensity = 20% |
| Wavelength of light = 3*d* |  |  |  |
| Wavelength of light = 1.5*d* |  |  |  |
| Wavelength of light = 0.75*d* |  |  |  |

The table below summarizes the main ideas of this tutorial. In the table, *λ* is the wavelength of the photons emitted by the lamp and *d* is the distance between the slits. You should double check that the way you completed the table in question 18 is consistent with the information presented below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Lamp intensity high | Lamp intensity intermediate | Lamp intensity low |
| *λ* significantly larger than *d* | “Which-path” info available for very few electrons;Interference pattern unchanged | “Which-path” info available for very few electrons;Interference pattern unchanged | “Which-path” info available for very few electrons;Interference pattern unchanged |
| *λ* comparable to *d* (but still larger than *d*) | “Which-path” info available for a large fraction of electrons;Interference pattern harder to discern, but still visible | “Which-path” info available for a significant fraction of electrons;Interference pattern only slightly harder to discern | “Which-path” info available for very few electrons;Interference pattern unchanged |
| *λ* significantly smaller than *d* | “Which-path” info available for most electrons;Interference pattern destroyed | “Which-path” info available for a large fraction of electrons;Interference pattern harder to discern, but still visible | “Which-path” info available for very few electrons;Interference pattern unchanged |

(Note: In this table, “large” fraction is considered to be larger than a “significant” fraction.)