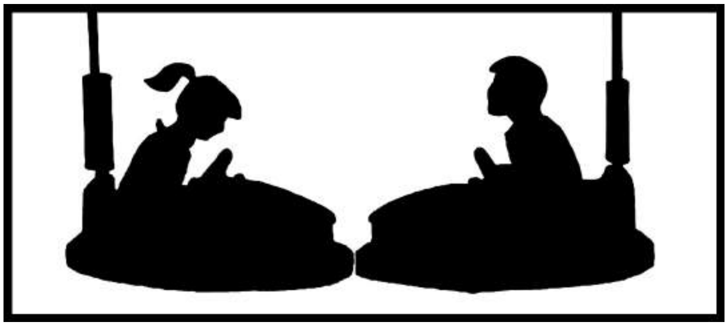
Name ________________________ Section _____________ Date ____________

UNIT 8: ONE-DIMENSIONAL COLLISIONS

A school bus full of students and a passenger car were both moving at approximately the same speed when they collided head on. The driver of the small car sustained serious injuries while the children escaped with only minor scratches. Newton's third law asserts that interaction forces are equal in magnitude and opposite in direction. If both vehicles experience the same *force magnitudes, why weren't the driver of the car and the students both injured seriously? When you complete this unit you should know whether or not Newton's third law applies to this type of collision or just to interactions between objects moving at a constant acceleration. You should also be able to explain why the driver of the passenger car sustained greater injuries.*[1]

UNIT 8: ONE-DIMENSIONAL COLLISIONS



In any system of bodies which act on each other, action and reaction, estimated by momentum gained and lost, balance each other according to the laws of equilibrium.

Jean de la Rond D'Alembert
18th Century

OBJECTIVES

1. To understand the definition of momentum and its vector nature as it applies to one-dimensional collisions.
2. To reformulate Newton's second law in terms of change in momentum, using the fact that Newton's "motion" is what we refer to as momentum.
3. To develop the concept of impulse to describe how forces act over time when an object undergoes a collision.
4. To use Newton's second law to develop a mathematical equation relating impulse and momentum change for any object experiencing a force.
5. To verify the mathematically-derived relationship between impulse and momentum experimentally.
6. To study the forces between objects that undergo collisions and other types of interactions in a short time period.
7. To formulate the Law of Conservation of Momentum as a theoretical consequence of Newton's laws and to verify it experimentally.

8.1. OVERVIEW

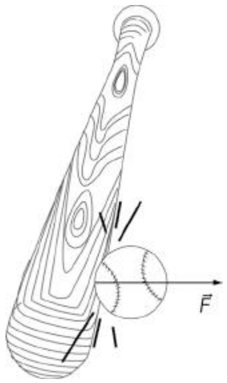


Fig. 8.1.

In this unit we will explore the forces of interaction between two or more objects and study the changes in motion that result from these interactions. We are especially interested in studying collisions and explosions in which interactions take place in fractions of a second or less. Early investigators spent a considerable amount of time trying to observe collisions and explosions, but they encountered difficulties.

This is not surprising, since the observation of the details of such phenomena requires the use of instrumentation that was not yet invented (such as the high speed camera). However, the principles of the outcomes of collisions were well understood by the late seventeenth century, when several leading European scientists (including Sir Isaac Newton) developed the concept of *quantity-of-motion* to describe both elastic collisions (in which objects bounce off each other) and inelastic collisions (in which objects stick together). These days we use the word *momentum* rather than *quantity-of-motion* in describing collisions and explosions.

We will begin our study of collisions by exploring the relationship between the forces experienced by an object and its momentum change. It can be shown mathematically from Newton's laws and experimentally from our own observations that the integral of force experienced by an object over time is equal to its change in momentum. This time-integral of force is defined as a special quantity called *impulse,* and the statement of equality between impulse and momentum change is known as the *impulse-momentum theorem*.

Next you will study the one-dimensional interaction forces between two colliding and exploding objects. By combining the results of this study with the impulse-momentum theorem, you can prove theoretically that momentum ought to be conserved in any interaction. It can be verified experimentally that whenever an object explodes or whenever two or more bodies collide, the momentum of the bodies before the event and their momentum after the event remain the same as long as no external force acts on them. At the conclusion of this study you will have the opportunity to use video analysis to verify the Law of Conservation of Momentum.

When the Law of Conservation of Momentum and the impulse-momentum theorem are applied to the study of collisions between two or more objects, physicists can learn about the interaction forces among them. There is no way to make direct measurements of the tiny forces of interaction between the various particles that are the fundamental building blocks of matter. Contemporary physicists working in accelerator laboratories bombard materials with tiny, rapidly moving particles and collect data on the momentum changes that occur during the collisions. This provides them with an indirect way of learning about fundamental forces of interaction. Those of you who continue the study of physics will revisit these relationships between momentum changes and forces many times.

MOMENTUM AND MOMENTUM CHANGE

8.2. DEFINING MOMENTUM

We are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet.

Let's start by predicting what will happen as a result of a simple one-dimensional collision. This should help you figure out how to define momentum to enable you to describe collisions in mathematical terms.

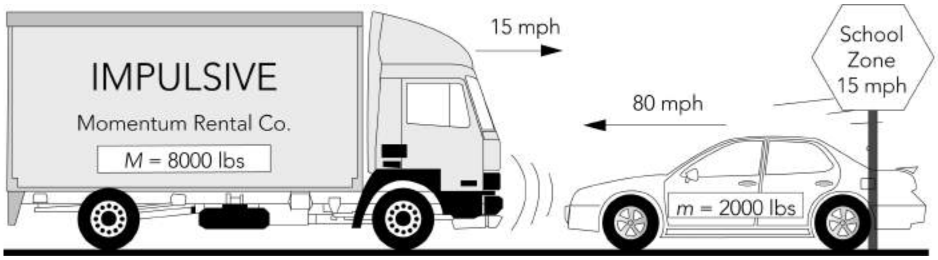


Fig. 8.2. An impending collision between two unequal masses.

It's early fall and you are driving along a two-lane highway in a rented moving van. It is full of all of your possessions, so you and the loaded truck were weighed in at 8000 lbs. You have just slowed down to 15 mph because you're in a school zone. It's a good thing you thought to do that because a group of first graders is starting to cross the road. Just as you pass the children you see a 2000 lb sports car in the oncoming lane heading straight for the children at about 80 mph. What a fool the driver is! A desperate thought crosses your mind. You figure that you just have time to swing into the oncoming lane and speed up a bit before making a head-on collision with the sports car. You want your truck and the sports car to crumple into a heap that sticks together and doesn't move. Can you save the children or is this just a suicidal act? For simulated observations of this situation you can use two carts of different masses set up to stick together in trial collisions. You will need:

- 2 dynamics carts
- 3 masses, 500 g (for one of the carts)
- 1 blob of clay (or velcro for sticky collisions)
- 1 cart ramp (or a smooth level surface)

Recommended group size:	3	Interactive demo OK?:	Y

8.2.1. Activity: Can You Stop the Car?

a. Predict how fast you would have to be going to completely stop the sports car. Explain the reasons for your prediction.

b. Try some head-on collisions with the carts of different masses to simulate the event. Describe some of your observations. What happens when the less massive cart is moving much faster than the more massive cart? Much slower? At about the same speed?

c. Based on your intuitive answer in part a. and your observations in part b., what mathematical definition might you use to describe the momentum (or motion) you would need to stop an oncoming vehicle traveling with a known mass and velocity?

Just to double-check your reasoning, you should have come to the conclusion that momentum can be defined by the vector equation

$$\vec{p} \equiv m\vec{v} \tag{8.1}$$

where the symbol ≡ means "defined as." The standard SI unit for momentum is $kg \cdot m/s$.

8.3. NEWTON'S SECOND LAW AS A FUNCTION OF MOMENTUM

Originally Newton did not use the concept of acceleration or velocity in his laws. Instead he used the term "motion," which he defined as the product of mass and velocity (a quantity we now call momentum). Let's examine a translation from Latin of Newton's first two laws (with some parenthetical changes for clarity).

Newton's First Two Laws of Motion[2,3]

1. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it.
2. The (rate of) change of motion is proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed.

The more familiar contemporary statement of the second law is that the net force on an object is the product of its mass and its acceleration where the direction of the force and of the resulting acceleration are the same. Newton's statement of the law and the more modern statement are mathematically equivalent, as you will show.

8.3.1. Activity: Re-expressing Newton's Second Law

a. Write down the contemporary mathematical expression for Newton's second law relating net force to mass and acceleration. Please use appropriate vector and summation signs.

b. Write down the definition of instantaneous acceleration in terms of the rate of change of velocity. Again, use vector signs.

c. It can be shown that if an object has a changing velocity and a constant mass, then

$$m\frac{d\vec{v}}{dt} = \frac{d\left(m\vec{v}\right)}{dt}$$

Explain why.

d. Show that $\Sigma\vec{F} = m\vec{a} = \dfrac{d\vec{p}}{dt}$.

e. Explain in detail why Newton's statement of the second law and the mathematical expression $\Sigma\vec{F} = d\vec{p}\,/\,dt$ are mathematically equivalent to each other.

8.4. MOMENTUM CHANGE AND COLLISION FORCES

What's Your Intuition?

You are sleeping in your sister's room while she is away at college. Your house is on fire and smoke is pouring into the partially open bedroom door. The room is so messy that you cannot get to the door. The only way to close the door is to throw either a blob of clay or a super ball at the door—there's not enough time to throw both.

8.4.1. Activity: What Packs the Biggest Wallop–A Clay Blob or a Super Ball?

Assuming that the clay blob and the super ball have the same mass, which would you throw to close the door—the clay blob (which will stick to the door) or the super ball (which will bounce back with almost the same speed it had before it collided with the door)? Give reasons for your choice, using any notions you already have or any new concepts developed in physics such as force, momentum, Newton's laws, etc. Remember, your life depends on it!

Observing the Wallop

Let's check out your intuition by using a ball on a pendulum to hit a wood block (a short length of 2 × 4). To do this you need to attach a bouncy super ball (known as a "live ball") to a string, and then pull the ball back just far enough to knock over the block when you let it go. Next you can hit the block in the same way with a clay blob (or "dead ball") attached to a string. We can associate the force exerted on the block by the balls with the force a thrown ball can exert on a door. We would like to investigate how these forces exerted by the live and dead balls are related to their momentum changes. To do these observations you'll need the following equipment:

- 1 live ball with hook (of mass *m*)
- 1 dead ball with hook (also of mass *m*)
- 1 rod clamp
- 1 right angle clamp
- 2 rods
- 1 string
- 1 ruler

Recommended group size:	4	Interactive demo OK?:	Y

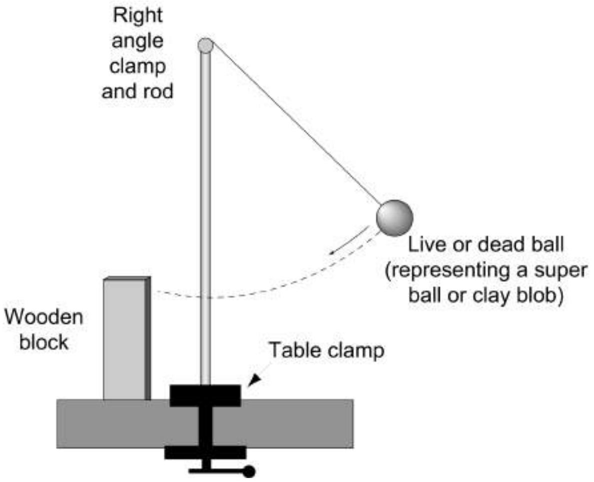


Fig. 8.3. Apparatus used to investigate the wallop delivered by a ball to a door.

8.4.2. Activity: Observing the Wallop of A Super Ball

a. Drop the live ball from a sufficient height so that as it swings down it just barely knocks the block over. Then replace the live ball with the dead ball (using a string of the same length). *Release the dead ball from exactly the same height*. Describe what happens.

b. How good was your intuition?

It would be nice to be able to use Newton's formulation of the second law of motion to find collision forces, but it is difficult to measure the rate of change of momentum during a rapid collision without special instruments. However, measuring the momenta of objects just before and just after a collision is usually not too difficult. This led scientists in the seventeenth and eighteenth centuries to concentrate on the overall changes in momentum that resulted from collisions. They then tried to relate changes in momentum to the forces experienced by an object during a collision. In the next activity you are going to explore the mathematics of calculating momentum changes.

8.4.3. Activity: Predicting Momentum Changes

Which object undergoes the most momentum change during the collision with a door—the clay blob or the super ball? Explain your reasoning carefully.

Formal Calculations of Momentum Changes

Let's check your reasoning with some formal calculations of the momentum changes for both inelastic and elastic collisions. This is a good review of the properties of one-dimensional vectors. Recall that momentum is defined as a vector *quantity* that has both magnitude and direction. Mathematically, momentum change is given by the equation

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 \tag{8.2}$$

where $\vec{p}_1$ is the momentum of the object just before and $\vec{p}_2$ is its momentum just after a collision.

8.4.4. Activity: Calculating 1D Momentum Changes

a. Suppose when the dead ball (or clay blob) hits the wooden block it sticks to the block. Assume the block remains standing and the dead ball has momentum just before it hits of $\vec{p}_1 = p_{1x}\hat{i}$ where $\hat{i}$ is a unit vector pointing along the *positive* x-axis. Express the final momentum of the dead ball in the same vector notation. **Reminder:** $\hat{i}$ and $\hat{j}$ represent unit vectors pointing along the x- and y-axes, respectively.

$\vec{p}_2 =$

b. What is the *change* in momentum of the clay blob as a result of its collision with the block? Use the same type of unit vector notation to express your answer.

$\Delta\vec{p} =$

c. Suppose that a live ball (or a super ball) hits the wooden block and "bounces" off it so that its speed just before and just after the bounce are the same. Also suppose that just before it bounces it has an initial momentum $\vec{p}_1 = p_{1x}\hat{i}$ where $\hat{i}$ is a unit vector pointing along the positive *x*-axis. What is the final momentum (after collision) of the ball in the same vector notation? **Hint:** Does the *p* vector after collision point along the +*x* or –*x*-axis?

$\vec{p}_2 =$

d. What is the *change* in momentum of the ball as a result of the collision? Use the same type of unit vector notation to express your result.

$\Delta\vec{p} =$

e. The answer is *not zero*. Why? How does this result compare with your prediction? Discuss this situation.

f. Suppose the mass of each ball is 20 g and that each ball hits the block at a speed of +0.30 m/s. Set up an *x*-axis and calculate the momentum just before the collision for each of the balls. Also calculate the momentum of the balls just after the collision. Use the following table to summarize your results. Use minus signs where appropriate.

Quantity	Units	Live ball	Dead ball
Mass, m			
Initial velocity, $\vec{v}_1$			
Initial momentum, $\vec{p}_1$			
Final velocity, $\vec{v}_2$			
Final momentum, $\vec{p}_2$			
Change in momentum, $\Delta\vec{p}$			

8.5. APPLYING NEWTON'S SECOND LAW TO THE COLLISION PROCESS

The Egg Toss

Suppose somebody tosses you a raw egg and you catch it. In physics jargon, one would say (in a very official tone of voice) that "the egg and the hand have undergone an inelastic collision." What is the relationship between the force you have to exert on the egg to stop it, the time it takes you to stop it, and the momentum change that the egg experiences? You ought to have some intuition about this matter. In more ordinary language, would you catch an egg slowly or fast? For this consideration you may want to use:

- 4 raw eggs (in shell)

Fig. 8.4.

Recommended group size:	All	Interactive demo OK?:	Y

8.5.1. Activity: Momentum Changes and Average Forces on an Egg: What's Your Intuition?

a. If you catch an egg of mass m that is heading toward your hand at speed v what is the magnitude of the momentum *change* that it undergoes?

b. Does the total momentum change differ if you catch the egg more slowly or is it the same?

c. Suppose the time you take to bring the egg to a stop is Δt. Would you rather catch the egg in such a way that Δt is small or large. Why?

d. What do you suspect might happen to the average force you exert on the egg while catching it when Δt is small?

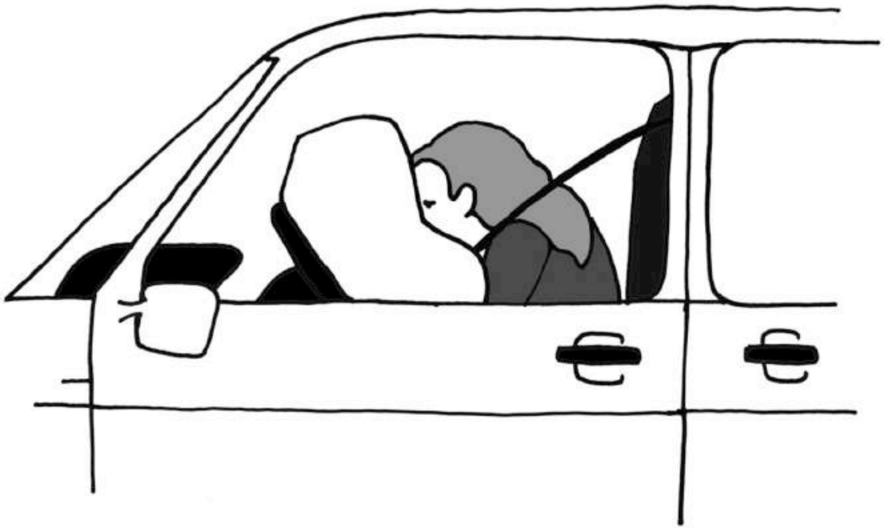


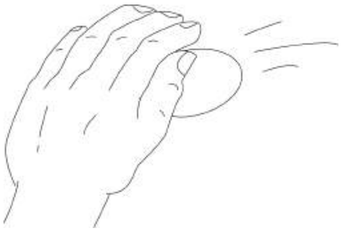
Fig. 8.5. If $\vec{F}\Delta t = \Delta\vec{p}$, then how would an air bag protect a driver?

Using Newton's Second Law to Describe Collisions

You can use Newton's second law to derive a mathematical relationship between momentum change, force, and collision times for objects. This derivation leads to the impulse-momentum theorem that we mentioned in the overview. Let's apply Newton's second law to the egg-catching scenario.

8.5.2. Activity: Force and Momentum Change

a. Sketch an arrow representing the magnitude and direction of the force exerted by your hand on the egg as you catch it.



b. Write the mathematical expression for Newton's second law in terms of the net force and the time rate of change of momentum of that object. (See Activity 8.3.1e for details.)

c. Explain why, if $\vec{F}$ is a constant during the collision lasting a time Δt, then

$$\frac{d\vec{p}}{dt} = \frac{\Delta\vec{p}}{\Delta t}$$

d. Show that for a constant force $\vec{F}$ the change in momentum is given by $\Delta\vec{p} = \vec{F}\Delta t$. Note that for a constant force, the term $\vec{F}\Delta t$ is known as the *impulse* given to one body by another.

IMPULSE, MOMENTUM, AND INTERACTIONS

8.6. THE IMPULSE-MOMENTUM THEOREM

Real collisions, like those between eggs and hands, a Nerf ball and a wall, or a falling ball and a platform scale are tricky to study because Δt is so small and the collision forces are not really constant over the time the colliding objects are in contact. Thus, we cannot calculate the impulse as $F\Delta t$. Before we study more realistic collision processes, let's redo the theory using a force that changes. In a collision, according to Newton's second law, the force exerted on a falling ball by the platform at any infinitesimally small instant in time is given by

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad (8.3)$$

To describe a general collision that takes place between an initial time t_1 and a final time t_2, we must take the integral of both sides of the equation with respect to time. This gives

$$\int_{t_1}^{t_2} \vec{F}\, dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \left(\vec{p}_2 - \vec{p}_1\right) = \Delta\vec{p} \qquad (8.4)$$

Impulse is a vector quantity *defined* by the equation

$$\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}\, dt \qquad (8.5)$$

By combining Equations 8.4 and 8.5 we can formulate the *impulse-momentum* theorem in which

$$\vec{J} = \Delta\vec{p} \qquad (8.6)$$

If you are not used to mathematical integrals and how to solve them yet, don't panic. If you have a fairly smooth graph of how the force F varies as a function of time, the *impulse integral can be calculated as the area under the F-t curve*.

Let's see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. In particular, let's play with a couple of objects that distort a lot during collisions:

- 1 foam ball
- 1 dynamics cart (with a plunger)

Recommended group size:	2	Interactive demo OK?:	N

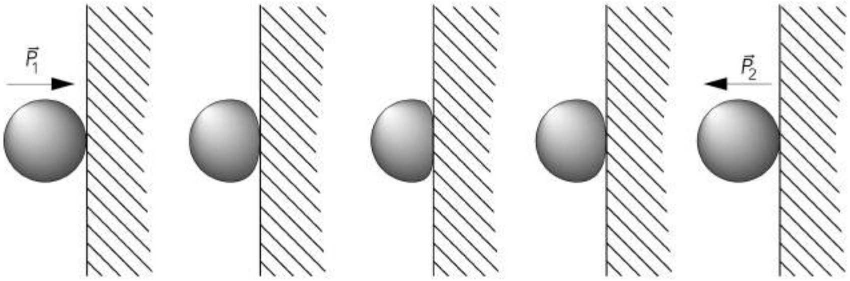


Fig. 8.6. A foam ball compressing and springing back during a collision.

8.6.1. Activity: Observing Collision Forces That Change

a. Suppose the cart with the spring-loaded plunger or a foam ball is barreling toward a wall and collides with it. If friction is neglected, what is the net force exerted on the object just before it starts to collide?

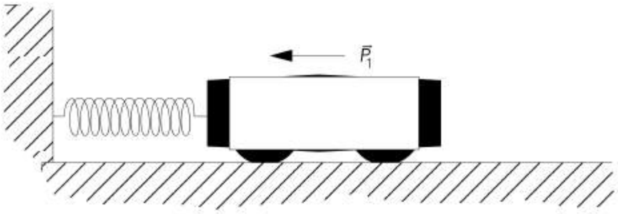


Fig. 8.7.

b. When will the magnitude of the force on the cart be a maximum? When the spring first starts to compress? While it is compressing? When it has a maximum compression? Explain.

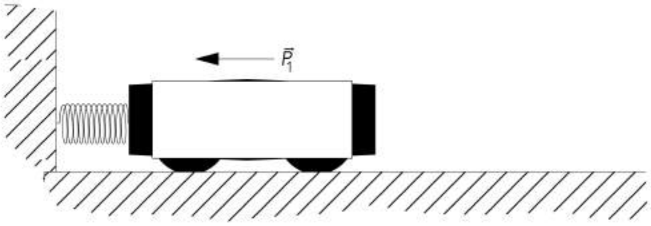
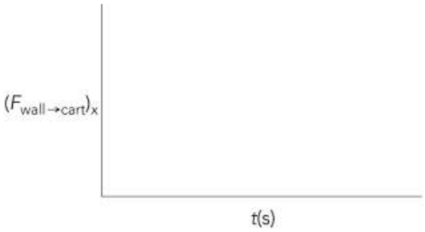


Fig. 8.8.

c. Watch the cart with its spring-loaded plunger collide with a wall several times. Roughly how long does the collision process take? Half a second? Less? Several seconds?

d. Remembering what you observed, attempt a rough sketch of the predicted shape of the curve describing the x-component of force the wall exerts on a moving cart during a purely horizontal collision.



8.7. VERIFICATION OF THE IMPULSE-MOMENTUM THEOREM

To verify the impulse-momentum theorem experimentally we must show that for an actual collision involving a single force on an object the equation

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}\, dt$$

holds, where the impulse integral can be calculated by finding the area under the curve of a graph of $(F_{\text{wall}\rightarrow\text{cart}})_x$ vs. t.

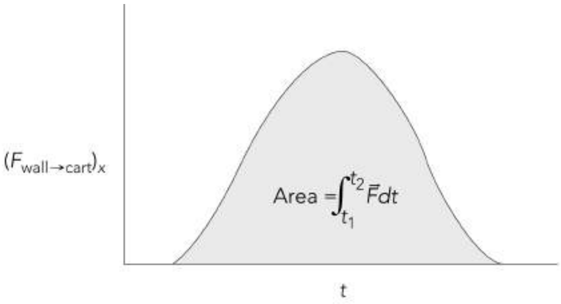


Fig. 8.9. Evaluating the impulse integral as the area under the graph of force on an object as a function of time as it undergoes a collision.

By using a computer-based laboratory system we can measure changes in force as a function of time during actual collisions. A computer-based laboratory force setup is shown for the case of a dynamics cart colliding with a force sensor in the following illustration.

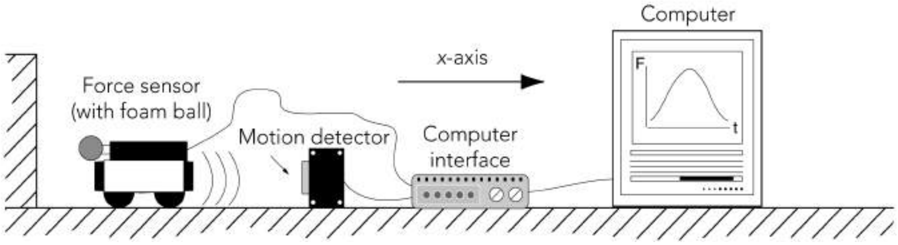


Fig. 8.10. Computer-based laboratory apparatus for measuring collision forces on a force sensor mounted on a cart and any object such as a wall or another cart, etc.

Your task is to see if you can set up a collision situation that will allow you to monitor the change of the net force on the cart as a function of time during the collision so that you can calculate the impulse

$$\vec{J}_{\text{cart}} \equiv \int_{t_1}^{t_2} \vec{F}_{\text{cart}}\, dt$$

experienced by the cart. At the same time, you must take measurements to allow you to determine the momentum before and after the collision and hence the momentum change of the object. You can then determine whether or not the impulse associated with the collision is equal (within the limits of experimental uncertainty) to the momentum change of the cart.

With a computer-based force sensing system that is capable of taking 80 (or preferably more) force readings a second, there are many ways to set up a verification experiment with the equipment available in a typical introductory physics laboratory. For this experiment we will focus on studying very gentle collisions between a dynamics cart with a force sensor firmly attached to it and another object. This is shown in Figure 8.10. The velocity of the cart-force system can be measured with a motion sensor attached to the computer interface unit.

Apparatus and supplies that you will need for this experiment include:

- 1 computer data acquisition system
- 1 force sensor
- 1 mass pan, 1.0 kg (if needed for calibration)
- 1 mass, 1 kg (if needed for calibration)
- 1 ultrasonic motion detector
- 1 electronic scale, 1000 g
- 1 ruler
- 1 dynamics cart
- 1 force sensor-to-cart holder (or tape)
- 1 foam ball (to slow down the collision time)

Recommended group size:	4	Interactive demo OK?:	Y

Since collisions can occur in a tenth of a second or less, your motion and force software must be set up carefully. For example, you will need to set up a rapid data collection rate for force and also set a trigger level so data are taken as soon as the collision starts. The sensor should be carefully calibrated and then set to zero with no force on it. **Warning:** Make sure you don't exceed the limitations of the force sensor you are using. (The maximum allowable force for common sensors is 50 N.)

8.7.1. Activity: An Impulse Experiment

a. Describe the measuring techniques and calculation methods you are using to determine the change in momentum of your object as a result of a collision with the force detector.

b. Do the experiment in which you determine both the velocity change and the impulse curve for the same gentle collision. *Remember, the maximum forces between the cart and the sensor must be less than 50 N.* After some practice, use the *integration* feature in the data acquisition software and figure out how to find the approximate value of the integral of the F vs. t curve for the time between the start and end times of your collision. Recall that the force of concern here is the force that the wall exerts on the cart. Explain what you did and list the value of the integral below.

c. Affix a small printout of your impulse curve in the space below.

d. Calculate the change in momentum of the cart during the collision and give its value in the space below.

e. Compare the change in momentum with the impulse—that is, with the area under the *F-t* curve. Does the impulse-momentum theorem seem valid within the limits of experimental uncertainty for your collision? Explain why or why not.

Note: So far we have only considered the situation where a *single* force acts on an object during a collision. If more than one force acts on an object then its momentum change is given by

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}^{\text{net}}\, dt$$

NEWTON'S LAWS AND MOMENTUM CONSERVATION

8.8. PREDICTING INTERACTION FORCES BETWEEN OBJECTS

In the last activities we focused our attention on the change in momentum that an object undergoes when it experiences a force that is extended over time (even if that time is very short!). Since interactions like collisions and explosions never involve just one object, we would like to turn our attention to the mutual forces of interaction between two or more objects. As usual, you will be asked to make some predictions about interaction forces and then be given the opportunity to test these predictions.

8.8.1. Activity: Predicting Interaction Forces

a. Suppose the masses of two objects are the same and that the objects are moving toward each other at the same speed.

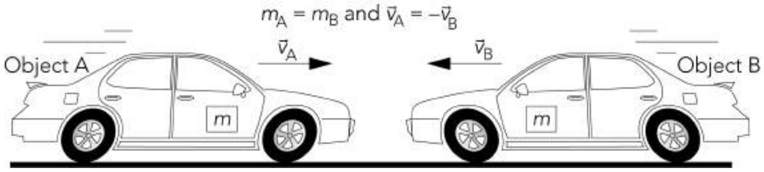


Fig. 8.11.

Predict the relative magnitudes of the forces between object 1 and object 2. Place a check next to your prediction.

______ Object A exerts more force on object B.
______ The objects exert the same force on each other.

______ Object B exerts more force on object A.

b. Suppose the masses of two objects are the same and that object A is moving toward object B, but object B is at rest.

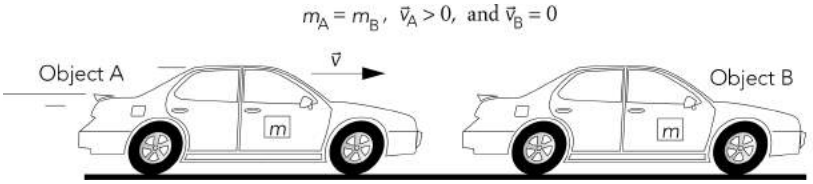


Fig. 8.12.

Predict the relative magnitudes of the forces between object A and object B. Place a check next to your prediction.

______ Object A exerts more force on object B.
______ The objects exert the same force on each other.
______ Object B exerts more force on object A.

c. Suppose the mass of object A is much less than that of object B and that it is pushing object B which has a dead motor so that both objects move in the same direction at speed *v*.

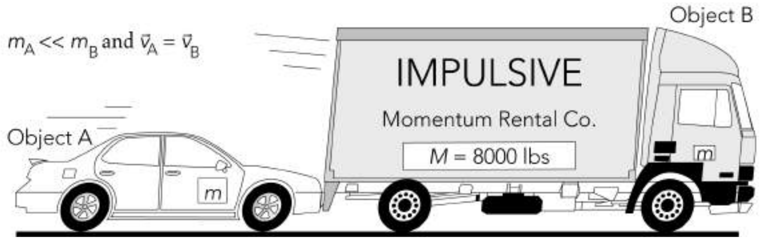


Fig. 8.13.

Predict the relative magnitudes of the forces between object A and object B. Place a check next to your prediction.

______ Object A exerts more force on object B.
______ The objects exert the same force on each other.
______ Object B exerts more force on object A.

d. Suppose the mass of object A is greater than that of object B and that the objects are moving toward each other at the same speed.

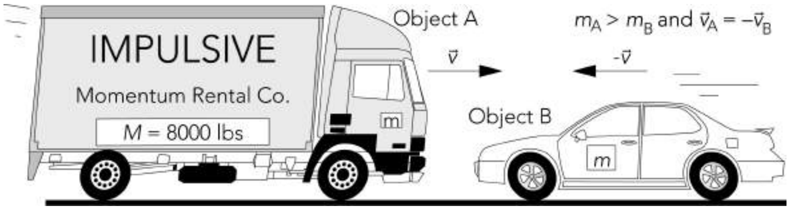


Fig. 8.14.

Predict the relative magnitudes of the forces between object A and object B. Place a check next to your prediction.

______ Object A exerts more force on object B.

______ The objects exert the same force on each other.

______ Object B exerts more force on object A.

e. Suppose the mass of object A is greater than that of object B and that object B is moving in the same direction as object A but not quite as fast.

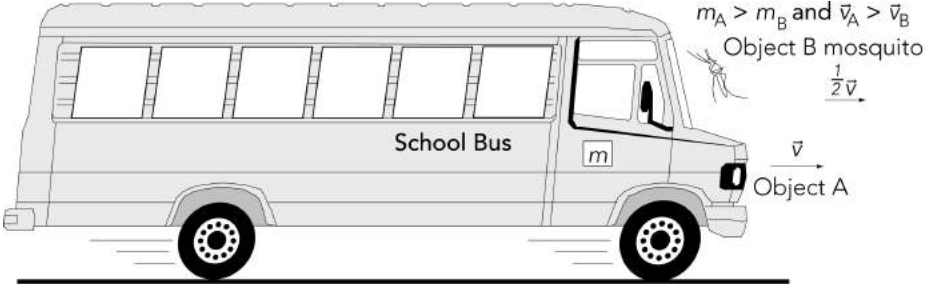


Fig. 8.15

Predict the relative magnitudes of the forces between object A and object B. Place a check next to your prediction.

______ Object A exerts more force on object B.

______ The objects exert the same force on each other.

______ Object B exerts more force on object A.

f. Suppose the mass of object A is greater than that of object B and that both objects are at rest until an explosion occurs.

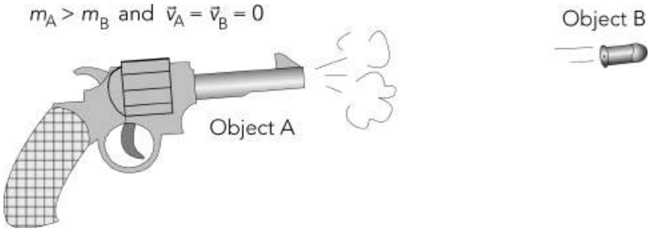


Fig. 8.16.

Predict the relative magnitudes of the forces between object A and object B. Place a check next to your prediction.

______ Object A exerts more force on object B.

______ The objects exert the same force on each other.

______ Object B exerts more force on object A.

g. Provide a summary of your predictions. What are the circumstances under which you predict that one object will exert more force on another object?

8.9. MEASURING MUTUAL FORCES OF INTERACTION

In order to test the predictions you made in the last activity you can study *gentle* collisions between two force sensors attached to carts. You can strap additional masses to one of the carts to increase its total mass so it has significantly more mass than the other. If a compression spring is available, you can set up an "explosion" between the two carts by compressing the spring between the force sensors on each cart and letting it go. To make these observations you will need the following equipment:

- 1 computer-based laboratory system
- 1 force software (for two force sensors)
- 2 force sensors with rubber stopper ends
- 1 mass, 1.0 kg (to calibrate the force sensors)
- 2 dynamics carts
- 3 masses, 500 g (to increase the mass of one cart)
- 1 ramp, 2 m (or level surface)

Recommended group size:	4	Interactive demo OK?:	Y

This apparatus should be set up as shown in the following diagram.

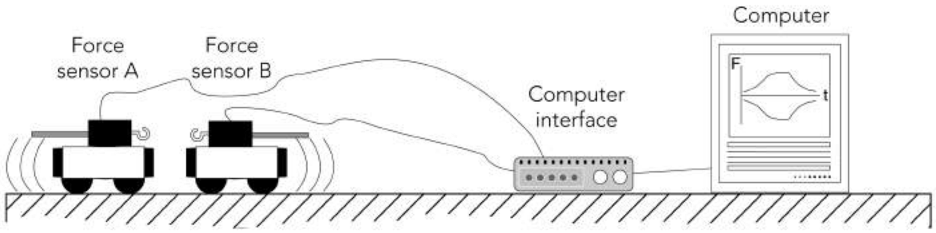


Fig. 8.17. Setup for reading two forces at once during a gentle collision or explosion.

Measuring Slow Interaction Forces

You can set up a computer-based laboratory system and the software needed to measure two mutual interaction forces for several seconds. For the next activity you should set the graph time scale to about 10 seconds.

8.9.1. Activity: Measuring Slow Forces

a. Play a gentle tug-of-war in which you *push* the ends of the two force sensors back and forth for about 10 seconds with your partner *using properly calibrated force sensors*. What do you observe about the mutual forces?

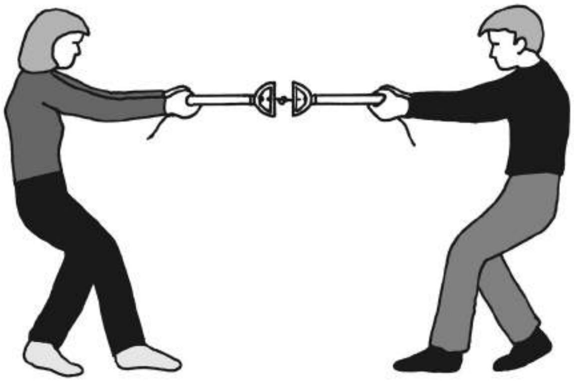


Fig. 8.18. Tug of war with force sensors.

b. Play a gentle tug-of-war in which you *pull* the ends of two force sensors back and forth for about 10 seconds with your partner *using properly calibrated force sensors*. What do you observe about the mutual forces?

Measuring Interaction Forces for Collisions

Now that you're warmed up to this two-force measurement technique, go ahead and try some different types of gentle collisions between two carts of different masses and initial velocities. Collisions can take place in about 0.05 seconds or less. When recording the interaction forces for a rapid collision, you should set the time scale to about 0.05 seconds, be sure to set the data collection rate for 1000 or more readings each second, and then set up a trigger mode so the readings start recording just as the collision starts.

8.9.2. Activity: Measuring Collision Forces

a. Use the two carts to explore various situations that correspond to the predictions you made about mutual forces. Your goal is to find out under what circumstances one object exerts more force on another object. Describe what you did in the space below and affix a printout of at least one of your graphs of force 1 vs. time and force 2 vs. time.

b. What can you conclude about forces of interactions during collisions? Under what circumstances does one object experience a different magnitude of force than another during a collision? How do the magnitudes and directions of the forces compare on a moment-by-moment basis in each case?

c. Do your conclusions have anything to do with Newton's third law?

d. How does the vector impulse due to object A acting on object B compare to the impulse of object B acting on object A in each case? Are they the same in magnitude or different? Do they have the same sign or a different sign?

$$\vec{J}_{\mathrm{A}\rightarrow\mathrm{B}} = \int \vec{F}_{\mathrm{A}\rightarrow\mathrm{B}}\, dt \text{ and } \vec{J}_{\mathrm{B}\rightarrow\mathrm{A}} = \int_{t_1}^{t_2} \vec{F}_{\mathrm{B}\rightarrow\mathrm{A}}\, dt$$

8.10. NEWTON'S LAWS AND MOMENTUM CONSERVATION

In your investigations of interaction forces, you should have found that the forces between two objects are equal in magnitude and opposite in sign on a moment-by-moment basis for all the interactions you studied. This is of course a testimonial to the seemingly universal applicability of Newton's third law to interactions between ordinary masses. You can combine the findings of the impulse-momentum theorem (which is really another form of Newton's second law since we derived it mathematically from the second law) to deduce the Law of Conservation of Momentum shown below.

Law of Conservation of Momentum

$$\Sigma\vec{p} = \vec{p}_{\mathrm{A}1} + \vec{p}_{\mathrm{B}1} = \vec{p}_{\mathrm{A}2} + \vec{p}_{\mathrm{B}2} \qquad (8.7)$$

where A refers to object A, B refers to object B, 1 refers to the momenta before collision, and 2 refers to the momenta after collision.

8.10.1. Activity: Deriving Momentum Conservation

a. What did you conclude in the last activity about the magnitude and sign of the impulse on object A due to object B and vice versa when two objects interact? (See Activity 8.9.2d.) In other words, how does $\vec{J}_{B\to A}$ compare to $\vec{J}_{A\to B}$?

$$\boxed{\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}\, dt}$$

b. Since you have already verified experimentally that the impulse-momentum theorem holds, what can you conclude about how the *change in momentum* of object A, $\Delta\vec{p}_A$, as a result of the interaction compares to the change in momentum of object B, $\Delta\vec{p}_B$, as a result of the interaction?

$$\boxed{\vec{J} = \Delta\vec{p}}$$

c. Use the definition of momentum change to show that the Law of Conservation of Momentum ought to hold for a collision, so that

$$\Sigma\vec{p} = \vec{p}_{A1} + \vec{p}_{B1} = \vec{p}_{A2} + \vec{p}_{B2} = \text{constant in time}$$

Verifying Momentum Conservation

In the next unit you will continue to study one- and two-dimensional collisions using momentum conservation. Right now you will attempt to verify the Law of Conservation of Momentum for a simple situation by using video analysis. To do this you will use a digital video movie in which two carts interact at a distance, with one transferring momentum to the other. You may not be able to finish this in class, but you can complete the project for homework.

To demonstrate and analyze a magnetic collision you will need:

- 2 dynamics carts with magnets attached
- 2 masses, 500 g (to place on a cart)
- 1 ramp
- 1 Video analysis software
- 1 digital movie entitled "PASCO021.MOV"

Recommended group size:	4	Interactive demo OK?:	Y

8.10.2. Activity: Verifying Momentum Conservation

a. Open the movie entitled “PASCO021.MOV.” By analyzing several of the video frames, take calibrated data for the positions of both carts as a function of time while they “collide.” Show your data table in the space below. You should only take data for three frames before and three frames after the frame in which the collision occurs. Since this is a horizontal 1D collision, the y-coordinates are of no interest.

b. Use your data to calculate the momenta of carts A and B during the three frames before the collision.

c. Use the data to calculate the momenta of carts A and B during the three frames after the collision.

d. Within the limits of experimental uncertainty, does momentum seem to be conserved (i.e., is the total momentum of the two-cart system the same before and after the collision)? **Note:** Frictional forces on the ramp may cause a 10% to 20% loss of momentum to the ramp.

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[1] Bill Smith/*The Sentinel,* Carlisle, PA.

[2] Newton, *Principia Mathematica,* Florian Cajori, Ed. (University of California Press, Berkeley, 1934). p. 13.

[3] L. W. Taylor, *Physics the Pioneer Science,* Vol. 1 (Dover, New York, 1959).